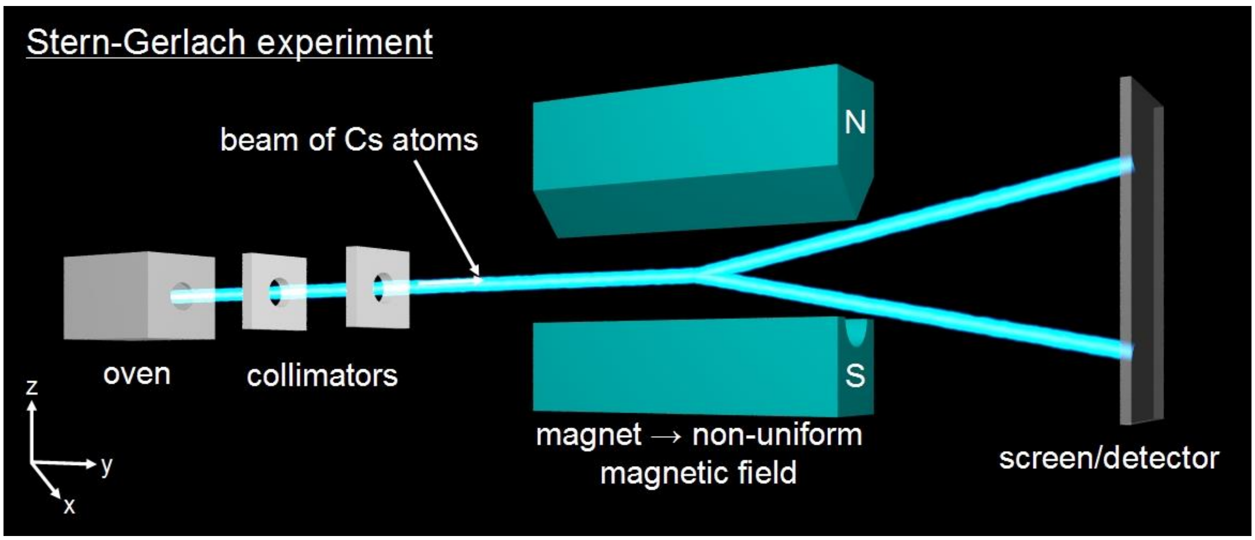
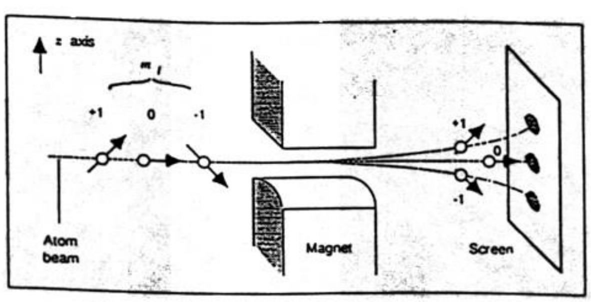
NOTES ON BELL'S THEOREM

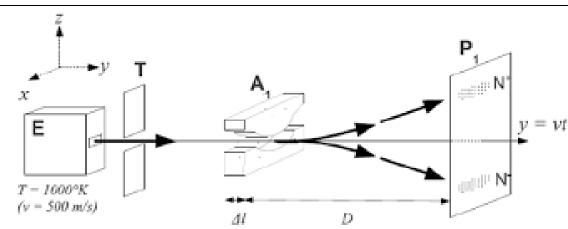
Bill Celmaster, January 2019 – revised

The Stern-Gerlach Experiment





Classically (**fake news)**, a spinning charge will be deflected in a continuous range depending on the **z-component of the spin vector** (**a.k.a 'angular momentum' vector)**

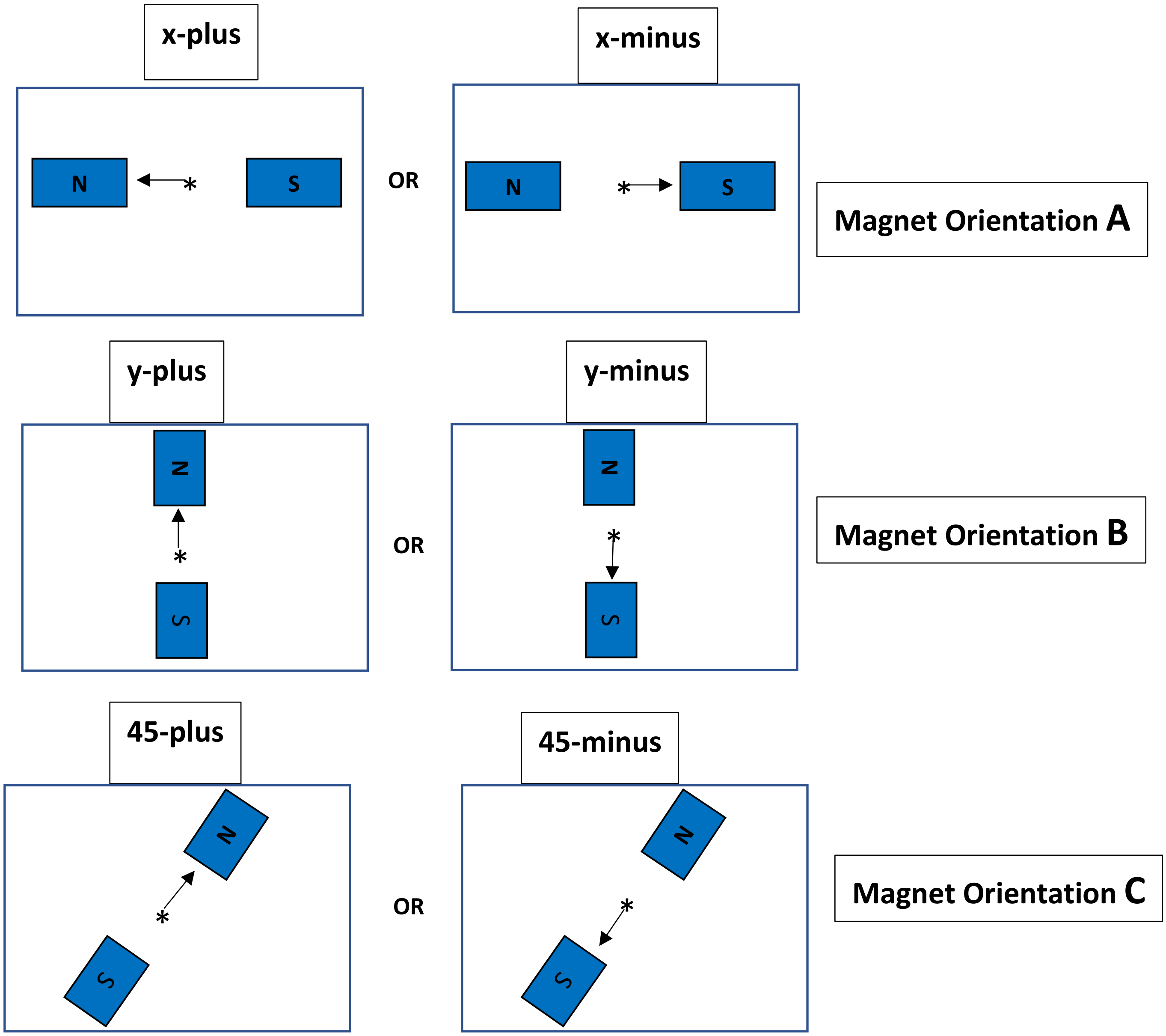


In the actual experiment, the charged particle is deflected in exactly one of two possible ways called "up" and "down". We interpret this as 'the particle has angular momentum of 'z-plus' or 'z-minus'.

Question for students of Lipoff Chapter 9: What is the value of "L" if there are only 2 possible states?

The above pictures show the magnets oriented in the z direction. If you rotate the magnets so that the N-S axis is along the y-axis instead of the z-axis, then deflections will also be along the y-axis. In the classical case ('fake news') we'd say that the deflection was related to the y-component of the spin vector. Again, in the actual experiment, only two deflections would be observed y-plus or y-minus.

Forward View. The particle goes <u>into</u> the paper (between the magnets)



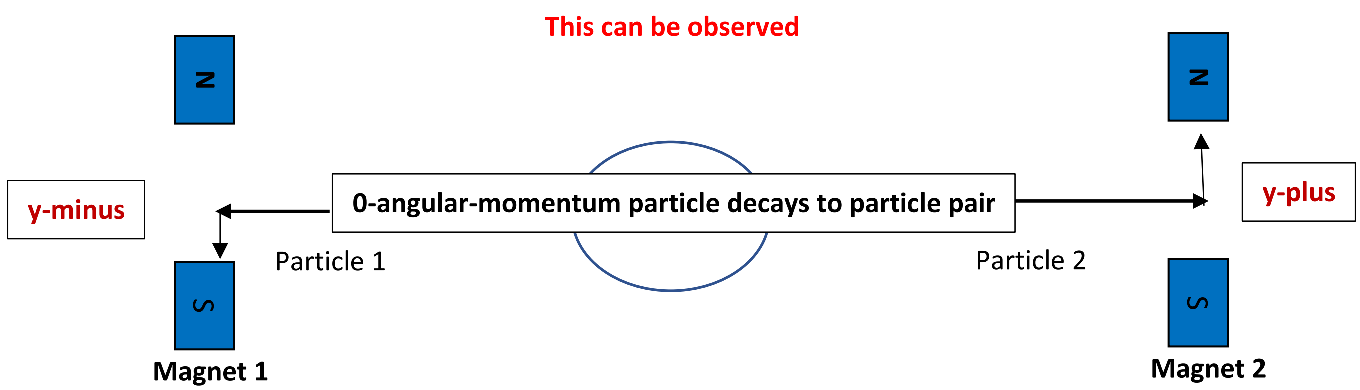
Of course, if we were to select other orientations, we would find the same thing – that there are exactly two spins.

The EPR experiment and Bell's theorem

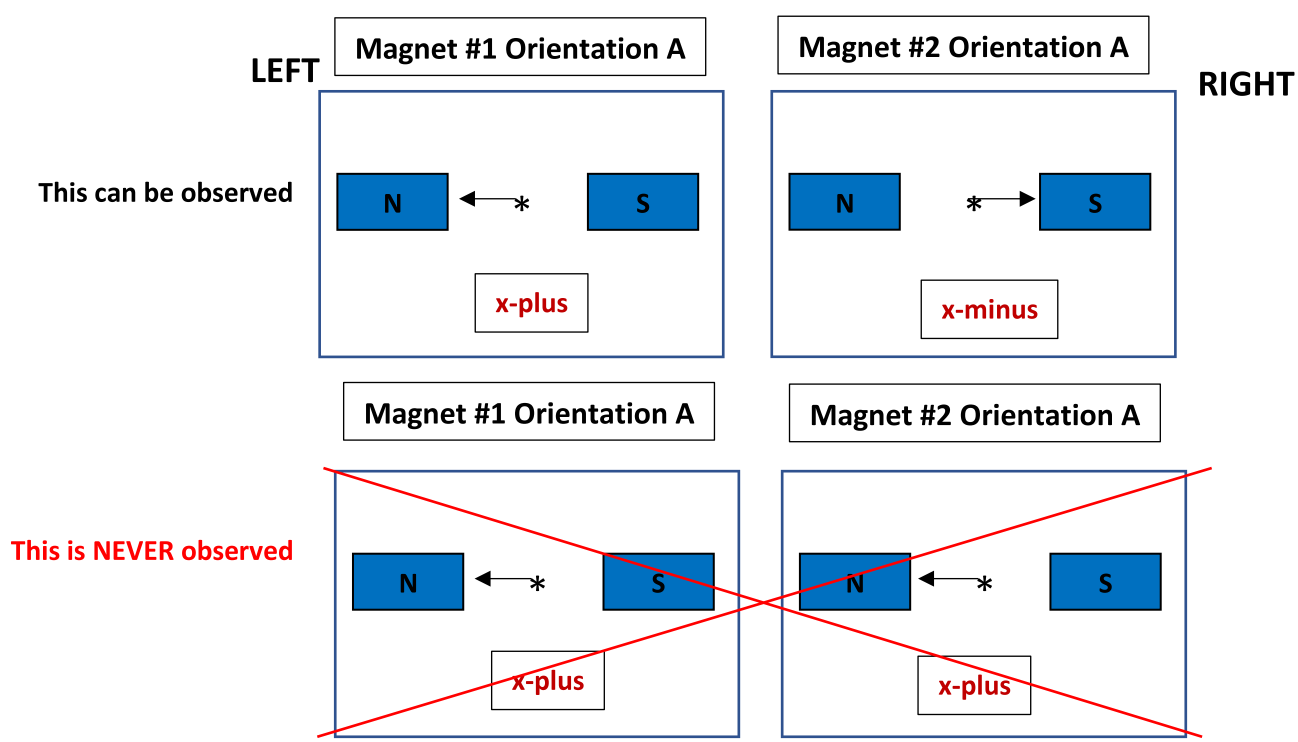
(This exposition follows a line of explanation which I believe was first put forward by Mermin.)

A key fact about angular momentum, is that it is a quantity which is conserved. (That turns out to be a consequence of the fact that the equations of motion are the same when you rotate your coordinate system.) Therefore, if you have a particle of angular momentum zero, which spontaneously decomposes (decays) into two oppositely moving particles, then **each of those particles must have equal but opposite values of angular momentum**.

Example with magnet orientation B

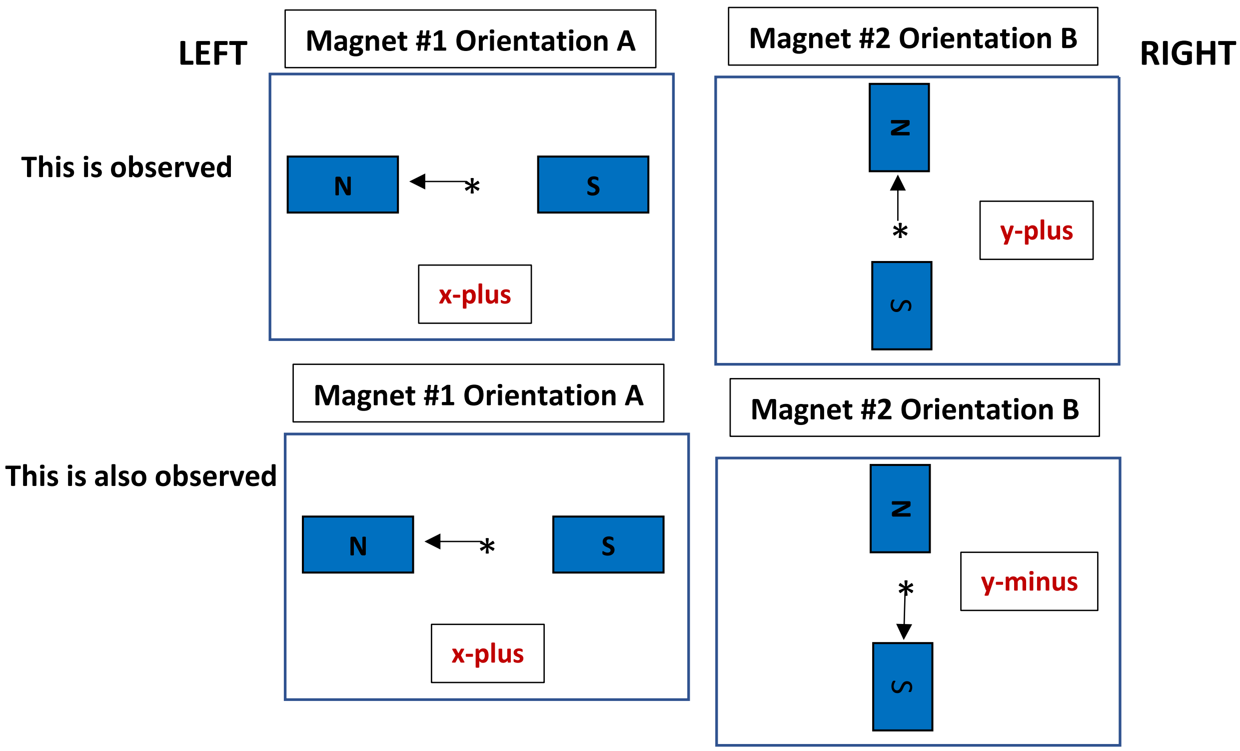


Example with magnet orientation A



In the above examples, **<u>if the left and right magnets both have the same orientation, then you ALWAYS observe opposite spins</u>** (plus/minus or minus/plus).

HOWEVER, if the two magnets have different orientations, then sometimes you'll observe opposite spins and sometimes you'll observe the same spin.



Interpretations of what's happening

1. **("Platonic"/ "realistic") Each particle 'has a property' that determines whether it is y-plus or y-minus, whether it is x-plus or x-minus, etc.** By conservation of angular momentum, if the left particle has the property of x-plus, then the right particle has the property of x-minus. Similarly, if the right particle has the property of y-minus (for example), then the left particle has the property of y-plus. **This is why, when the two magnets are oriented the same way, they will always detect opposite spins …** ***no matter how far apart the two magnets are!***
2. **("Instrumental") The particle does <u>not</u> have a property that determines, in advance, the direction of deflection when passing through magnets with orientation A (or any other orientation). The deflection-direction (e.g. 'x-plus' versus 'x-minus') is determined during the process of measurement.** But somehow there is ***a mysterious way in which angular momentum is conserved*** so that the two far-apart magnets – when oriented in identical directions – detect opposite spin. (<u>Warning: there can be no communication between the left and right detection events.</u>)

Next: Random Orientations

- **Assume there is a mechanism on each magnet, which randomly picks the orientation.**
- **Assume the magnets are re-oriented while the particle is travelling, and that the magnets are so far apart that the particles reach their magnets before any signal could inform magnet 2 of the orientation of magnet 1 (and vice versa). In other words, the magnet orientations can be set independently, without any possibility that magnet 1 can interfere with particle 2 prior to measurement and vice versa.**

All Possible Observations for Different Orientations

Magnet 1 Orientation	Magnet 2 Orientation	Observed Spins (deflections) (conservation of angular momentum)		Are the 2 spins different?
		Left-hand particle	Right-hand particle	
A	A	x-plus	x-minus	Y
A	A	x-minus	x-plus	Y
A	B	x-plus	y-plus	N
A	B	x-plus	y-minus	Y
A	B	x-minus	y-plus	Y
A	B	x-minus	y-minus	N
A	C	x-plus	45-plus	N
A	C	x-plus	45-minus	Y
A	C	x-minus	45-plus	Y
A	C	x-minus	45-minus	N
B	A	y-plus	x-plus	N
B	A	y-plus	x-minus	Y
B	A	y-minus	x-plus	Y
B	A	y-minus	x-minus	N
B	B	y-plus	y-minus	Y
B	B	y-minus	y-plus	Y
B	C	y-plus	45-plus	N
B	C	y-plus	45-minus	Y
B	C	y-minus	45-plus	Y
B	C	y-minus	45-minus	N
C	A	45-plus	x-minus	Y
C	A	45-plus	x-plus	N
C	A	45-minus	x-plus	Y
C	A	45-minus	x-minus	N
C	C	45-minus	45-plus	Y
C	C	45-plus	45-minus	Y
C	B	45-plus	y-plus	N
C	B	45-plus	y-minus	Y
C	B	45-minus	y-plus	Y
C	B	45-minus	y-minus	N

The only combinations not observed are two equal orientations with RH column = N

One might think that all above results are possible with equal likelihood. But, amazingly, that isn't what we will predict in the 'platonic interpretation'.

What do we predict?

- Our prediction will concern the following: For each pair of particles and each randomly chosen pair of magnet orientations, record whether the spins correlate (Y) or anti-correlate (N).
- In the Platonic interpretation, the left-particle has spin properties (i.e. pre-ordained spin values of 'plus' or 'minus') including x, y and '45' spins, and the right-particle has the opposite. **Furthermore,** since there can be no communication, the **measurement of the left particle cannot influence the measurement of the right particle.**
- Consider an example: Left-particle has properties x-plus, y-plus, 45-minus. So right-particle properties are x-minus, y-minus and 45-plus. If magnet 1 had A-orientation and magnet 2 had B-orientation, spin-1 is 'plus' and spin-2 is 'minus' (x-plus; y-minus). The correlation is "Y".
- Another example: As before, left-particle has properties x-plus, y-plus, 45-minus. So as before, right-particle properties must be x-minus, y-minus and 45-plus. If magnet 1 had A-orientation and magnet 2 had C-orientation, **what are the 2 spin properties for the left and right particle and what is the correlation value?**

The question we will ask is this: IF the left particle (and therefore the right particle) had the properties in the above example, what is the **likelihood** that the correlation-value is Y? Remember, all pairs of magnet orientations are equally likely.

	Magnet 2 Orientation		
Magnet 1 Orientation	**A**	**B**	**C**
A	Y	Y	N
B	Y	Y	N
C	N	N	Y

So, we conclude there is a likelihood of 5/9 that we measure the correlation-value "Y".

Now you try it! Assume this time, that the left-hand particle properties are x-minus, y-plus, 45-minus. What are the right-hand particle properties? Fill in the matrix.

	Magnet 2 Orientation		
Magnet 1 Orientation	**A**	**B**	**C**
A	**?**	**?**	**?**
B	**?**	**?**	**?**
C	**?**	**?**	**?**

What is the likelihood that we measure the correlation-value "Y"?

Since there is nothing special about one orientation versus another, it should be easy to see that **as long as the left particle has the 2 spin-properties that are equal to one another, and the remaining spin-property is different, there is a likelihood of 5/9 that we measure the correlation-value "Y"!!!!!**

The **only** other possibility is that all left-particle spin properties are identical. For example, x-plus, y-plus, 45-plus. In that case, all right-properties are opposite and therefore ALL correlation-values will be "Y".

	Magnet 2 Orientation		
Magnet 1 Orientation	**A**	**B**	**C**
A	Y	Y	Y
B	Y	Y	Y
C	Y	Y	Y

When the left particle has all 3 spin properties equal to one another, then there is a likelihood of 1 that we measure the correlation-value "Y"!!!!!

We have now covered all possibilities.

Bell's Theorem

Assuming the platonic interpretation…

For any pair of particles and therefore for beams of particle-pairs, P(Y) ≥ 5/9.

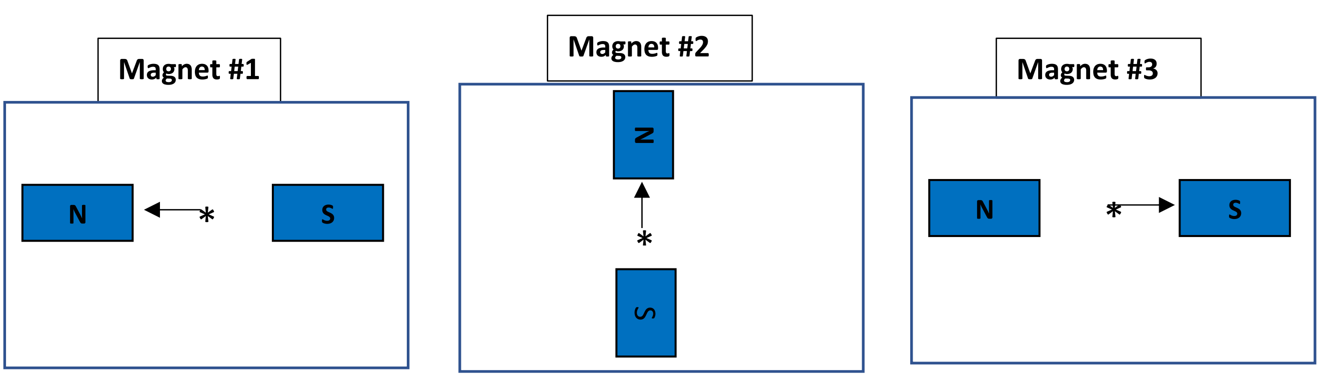
These experiments are difficult and started being done in the 1970's. I don't know if any were done with Stern-Gerlach style experiments. Most are done with photons but using very similar principles. Experimental loopholes of one sort and the other are still being plugged but physicists now have very high confidence in the results.

Experimentally, the inequality is violated! The platonic interpretation is wrong. What is truly amazing is that, due to Bell's theorem, it is possible to show experimentally that it is wrong.

Also, note that the measurement on the left can't affect the measurement on the right. So, these results are hard to reconcile with the idea that the origins of Heisenberg's uncertainty principle have to do with interference of observer with observed. It is also hard to reconcile these observations with any simple mental picture of the so-called *collapse of the wavefunction*, where one imagines that the word 'collapse' refers to an event that happens at a particular time [we could have guessed this was tricky, since the wavefunction is typically spread out in space, and according to relativity, there isn't any meaning to simultaneity at two different points in space.]

Some History

Historically, physicists first did experiments where a single particle first went through one magnet, then through a second one, and finally through a third one (I don't know for a fact that this was an experiment they did, but there were plenty of experiments like it which led to the same conclusions). When all magnets were aligned the same way, then the deflections were all in the same direction. However, if the first and third magnets were oriented the same way (for example A), and the middle magnet were oriented differently (for example B), then the deflections from the first and third magnet could sometimes be different. (See the picture below).



In the realist/platonic interpretation, this phenomenon was attributed to the idea that there was some fundamental interference between the second measurement (magnet #2; y-spin) and the x-spin property.

- The platonists/realists say that the act of measurement changes the pre-ordained deflection-value (i.e. spin-property) of what is being measured.
- It is often explained by the instrumentalists by saying that the act of measurement causes the particle to acquire a deflection-value (spin).

Notice that in both the platonic and instrumental interpretations, we come to the conclusion that it's impossible to "simultaneously" measure A and B spins. This is referred to as the Heisenberg Uncertainty Principle.

During the early days of quantum mechanics (1905 – 1920's), physicists were busy trying to figure out detailed rules and the 'interpretations' weren't especially important. Some of the biggest progress was made after 1915 by the school of thought (the so-called Copenhagen School) of Heisenberg and Bohr, and they adopted the attitude that "if you can't measure it, you can't talk about it". This reflected a stream of philosophy (instrumentalism) that broke with the platonic idea that all our perceptions reflect some underlying 'reality'. The key significance of all this, is that the Copenhagen School felt free to develop a coherent mathematics of quantum phenomena which ignored the question of "what's going on under the covers". By contrast, Einstein and his followers (the 'realist' school) believed that Quantum Mechanics was 'incomplete' and that more research was necessary to uncover the mechanisms responsible for the underlying 'properties'. During this period, no-one was able to conclusively demonstrate their point of view and, importantly, those views made no difference other than in metaphysical discussions. [side-comment for another time: some similar kind of view-shift is underway right now vis-à-vis the so-called anthropic principle and its relevance to a multiverse interpretation of reality]

And then what happened?

In the mid-30's, Einstein together with collaborators Podalsky and Rosen (EPR) noticed that a particular kind of experiment would have a very peculiar consequence if the platonic view wasn't right. They wrote a paper describing this experiment and came to the conclusion that the Copenhagen interpretation would, if correct, imply certain experimental correlations between events that were too far apart to allow any communication between them. Einstein referred to this as "Spooky Action at a Distance" (and he didn't like it!) EPR DID NOT CONCLUDE THAT THE PLATONIC INTERPRETATION WAS RIGHT, BUT ONLY CONCLUDED THAT IF IT WERE WRONG, THEN THERE WOULD BE SPOOKY ACTION AT A DISTANCE.

John Bell's theorem essentially stated, **"In the EPR experiment, if you end up observing spooky action at a distance, then the Platonic interpretation is wrong."** **And that of course is what happened.**