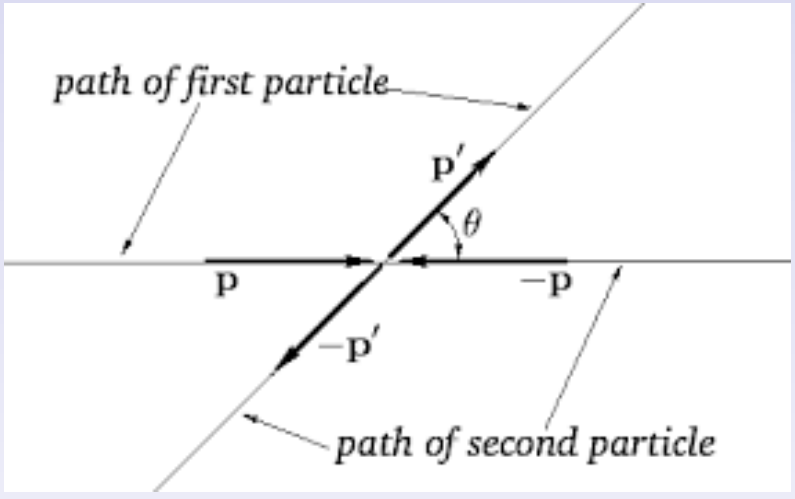
Physics probabilities from Green functions – Bill Celmaster, November 2020

- Example scattering experiment with *scattering amplitude* $\langle p_1^{out}, p_2^{out} | S | p_1^{in}, p_2^{in} \rangle$



- The scattering amplitude is computed from the Green function.
- Green function (one field example):

$$G(x_1, ..., x_n) = \lim_{\epsilon \to 0} \frac{\int [\mathcal{D}\phi] \phi(x_1) ... \phi(x_n) e^{i(S[\phi] + i\epsilon)}}{\int \mathcal{D}\phi e^{i(S[\phi] + i\epsilon)}} \tag{1}$$

Computing the Green function by numerical simulation of path integrals – example

Simplify above to one dimension (time) and replace ϕ by q.

$$G(t_a, t_b) = \lim_{\Delta t_i \to 0} \frac{\int (\prod_{t_i}^{\infty} dq_{t_i})(q_{t_a} q_{t_b} \prod_{t_j} e^{i(L(q(t_j), q(t_{j+1})) + i\epsilon)})}{\int (\prod_{t_i}^{\infty} dq_{t_i}) \prod_{t_j} e^{i(L(q(t_j), q(t_{j+1})) + i\epsilon)}}$$

Numerical approximation of path integral of F

Multi-dimensional integral of F, integrated (numerically) along each line.

$$\sum_{q_{t_0}} \sum_{q_{t_1}} \sum_{q_{t_2}} \sum_{q_{t_3}} \sum_{q_{t_4}} \sum_{q_{t_5}} F(q_{t_5}, q_{t_4}, q_{t_3}, q_{t_2}, q_{t_1}, q_{t_0})$$

$\{q_{t_i}\}$ is called the lattice