What Monsters Might Be Lurking There?

Testing the anomalous magnetic moment of the muon

Bill Celmaster, April 2021

‘WHAT MONSTERS MIGHT BE LURKING THERE?’

CHRIS POLLY,
Fermilab physicist, speaking at an April 7 news conference on experiments with magnetic fields that suggest the tiny particles known as muons do not always obey known laws of physics, a discovery scientists believe could change our understanding of the evolution of our universe

‘I’ve ran from myself for a long time. I’ve hated myself for a long time.’

From Time Magazine Apr 26 – May 3 2021

Outline

- Classical theory of charged particle in a magnetic field
- Magnetic moment
- g=2: Spin magnetic moment according to the Dirac equation
- Measuring the magnetic moment of an electron
- What is a muon?
- Fermilab
- Measuring the magnetic moment of a muon
- How are corrections to g=2 (anomalous magnetic moment) predicted?
    - Feynman diagrams, Nobel prizes and the electron a.m.m.
    - What’s different about the a.m.m. of the muon – strong interactions?
- Dispersion and lattice predictions compared to Fermilab results
    - Discovery, tension and marketing

Classical theory of charged particle in EM field

- $F = e(E + \frac{1}{c}\, v \; \times B)$
- This follows from Euler-Lagrange equations for $\mathcal{L} = \frac{m\dot{\boldsymbol{q}}^2}{2} + e(\frac{\dot{\boldsymbol{q}}\cdot\boldsymbol{A}}{c} - \varphi)$
  - The dynamic variables are the particle coordinate and velocity q and $\dot{q}$. The EM fields are 'external' and not influenced by the particle.
- Leads to Hamiltonian $H = \frac{1}{2m}(\boldsymbol{p} - \frac{e}{c}\boldsymbol{A})^2 + e\varphi$
- When **B** is constant, you can write $\boldsymbol{A} = \frac{1}{2}\boldsymbol{r} \times \boldsymbol{B}$ and then

$$H = \frac{p^2}{2m} - \frac{e}{2mc}\boldsymbol{L} \cdot \boldsymbol{B} + \;\ldots$$

$(\boldsymbol{L} = \boldsymbol{r} \times \boldsymbol{p})$

Magnetic moment

- The term $\frac{e}{2mc}\boldsymbol{L}$ in this equation is called *the magnetic moment,* ***m****,* of the charged particle.
- This is typically associated with a magnetic dipole, which when acted on by a magnetic field, experiences a torque $\boldsymbol{m} \times \boldsymbol{B}$.
- The relationship of **m** to $\frac{e}{2mc}\boldsymbol{L}$ comes from considering a small loop of current, I. This current causes a magnetic field, and the associated magnetic (dipole) moment is |**m** |**=** IA where A is the area enclosed by the loop.

Dirac equation

$$H = \frac{\boldsymbol{p}^2}{2m} - \frac{e}{2mc}\boldsymbol{L}\cdot\boldsymbol{B} - \frac{ge}{2mc}\boldsymbol{S}\cdot\boldsymbol{B} + \ \ldots$$

($\boldsymbol{S} = 2\hbar\boldsymbol{\sigma}$, is *the spin*)

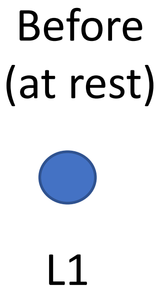
- Before Dirac, the spin term had been hypothesized to explain atomic energy levels – the so-called anomalous Zeeman effect.
- **g** was a parameter – origin unknown – called *the Lande g-factor* and measured to be ≈2.
- The Bohr magneton is defined as $\mu_B = \frac{e\hbar}{2mc}$
- Dirac's derivation showed $g = 2$.
- Higher order corrections due to dynamics of electromagnetic field: called anomalous magnetic moment: $a = \frac{g-2}{2}$.

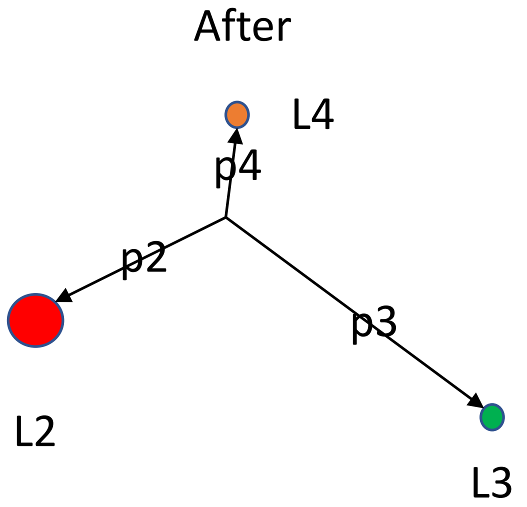
What is a muon?

- μ is just like an electron but 207 times heavier.
- Like the electron, it is associated with its own (almost massless) neutrino ν.
- $\binom{e^-}{\nu_e}$ $\binom{\mu^-}{\nu_\mu}$. Each entry has lepton number + 1.
- $\binom{e^+}{\bar{\nu}_e}$ $\binom{\mu^+}{\bar{\nu}_\mu}$. These are the antiparticles. Lepton number -1.
- In reactions, lepton number is conserved (in all experiments so far).
- Also, energy is conserved. Particle energy is $E = \sqrt{(mc^2)^2 + p^2}$. (p is momentum)

How does a muon differ from an electron?

- The electron is stable. A muon isn't. Why not?





- Lepton number is conserved so L1=L2+L3+L4
- Energy is conserved so E1 = E2+E3+E4

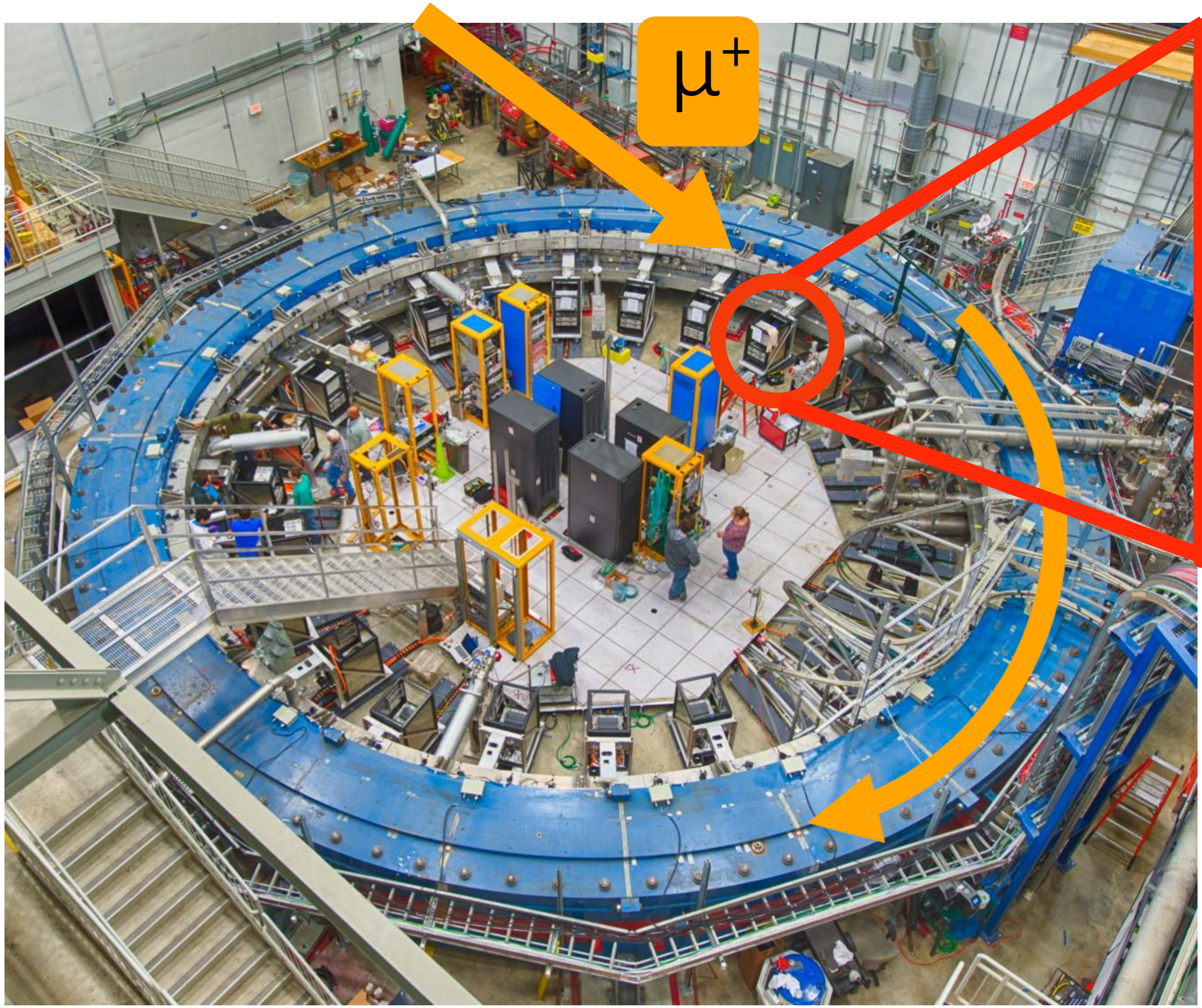
If particle 1 is an electron, then L1 = 1. Try L2 = 1, L3 = 1, L4 = -1.   Particle 2 is electron or muon. If muon, then E2 > E1.
If electron, then all other masses must be 0 and all momenta must be 0. So electron can't decay!

If particle 1 is a muon, then it can decay into an electron, a neutrino and an antineutrino. Or two electrons and a positron etc.
A muon decays so doesn't hang around in nature! It has to be produced in reactions. Much harder to measure anything.

FERMILAB


EMMERT

Muon Beam Injection

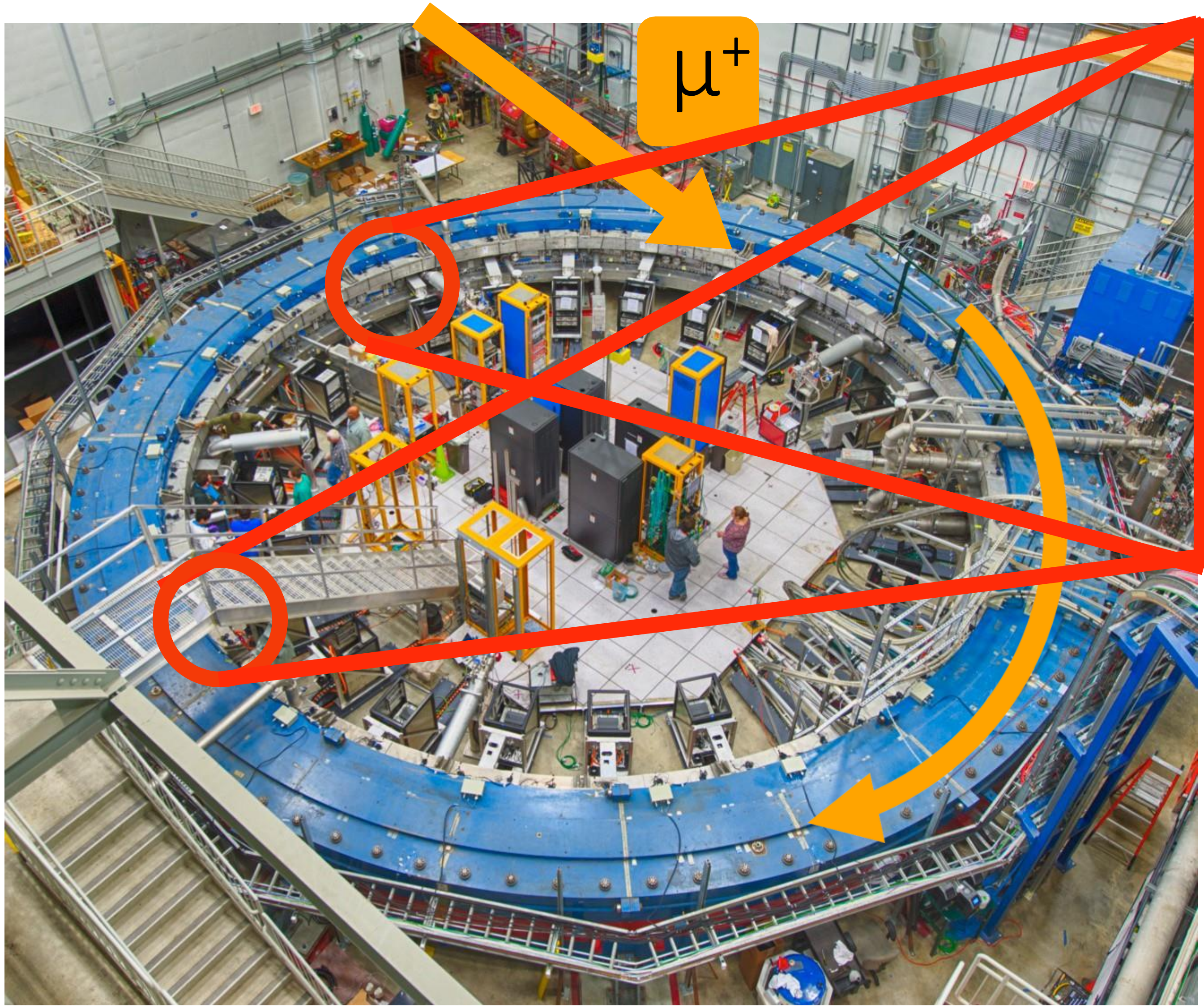


24 segmented PbF_2 crystal calorimeters

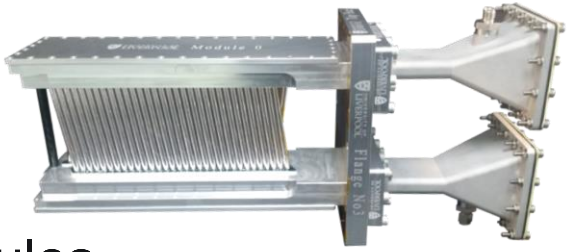
- Each crystal array of 6 x 9 PbF_2 crystals
  - 2.5 x 2.5 cm^2 x 14 cm ($15X_0$)
- Readout by SiPMs to 800 MHz WFDs
  (1296 channels in total)

This slide and some others are taken from a Joe Price presentation

Muon Beam Injection



2 Tracking stations



- Each contain 8 modules
- 128 gas filled straws in each module
- Traceback postrons to their decay point

Measurement Principle

- Inject polarized muon beam into magnetic storage ring
- Measure **difference** between spin precession and cyclotron frequencies

Different frequencies imply different energies measured in calorimeters.

- If g = 2, $\omega_a = 0$
- $g \neq 2$, $\omega_a \cong (e/m_\mu)a_\mu B$

Spin precession freq.

$$\omega_s = \frac{geB}{2mc} + (1-\gamma)\frac{eB}{\gamma mc}$$

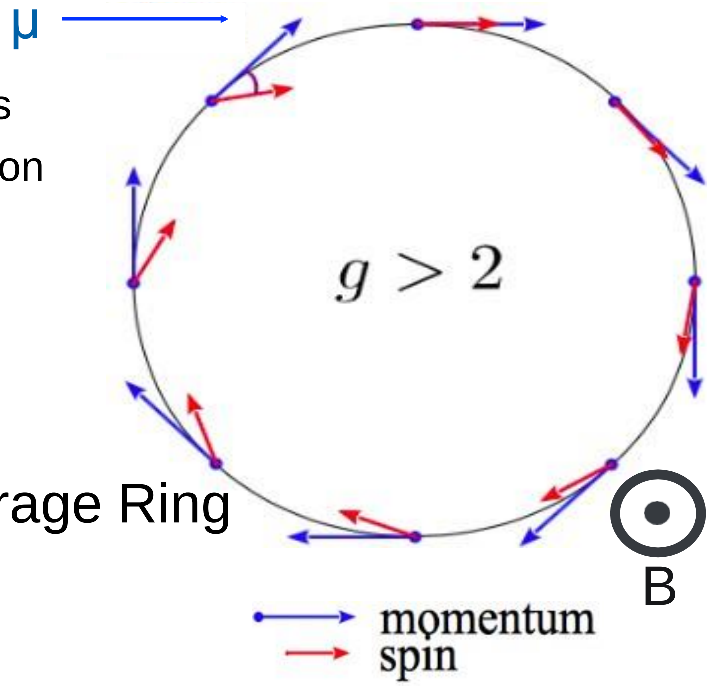
Larmor precession

Thomas precession

Cyclotron freq.

$$\omega_c = \frac{eB}{\gamma mc}$$

$$\omega_a = \omega_s - \omega_c = a_\mu \frac{eB}{mc}$$



$$a_\mu = \frac{\omega_a}{\tilde{\omega}_p}\frac{\mu_p}{\mu_e}\frac{m_\mu}{m_e}\frac{g_e}{2}$$

3 ppb 22 ppb 0.3 ppt

Rev. Mod. Phys. **88**, 035009 (2016)

- We measure ω_a and ω_p separately
- Aiming for 70 ppb precision on each (systematic)
- **Target: δa_μ(syst) = 140 ppb; factor of 4 improvement over BNL**

See the Wikipedia article on the gyromagnetic ratio.

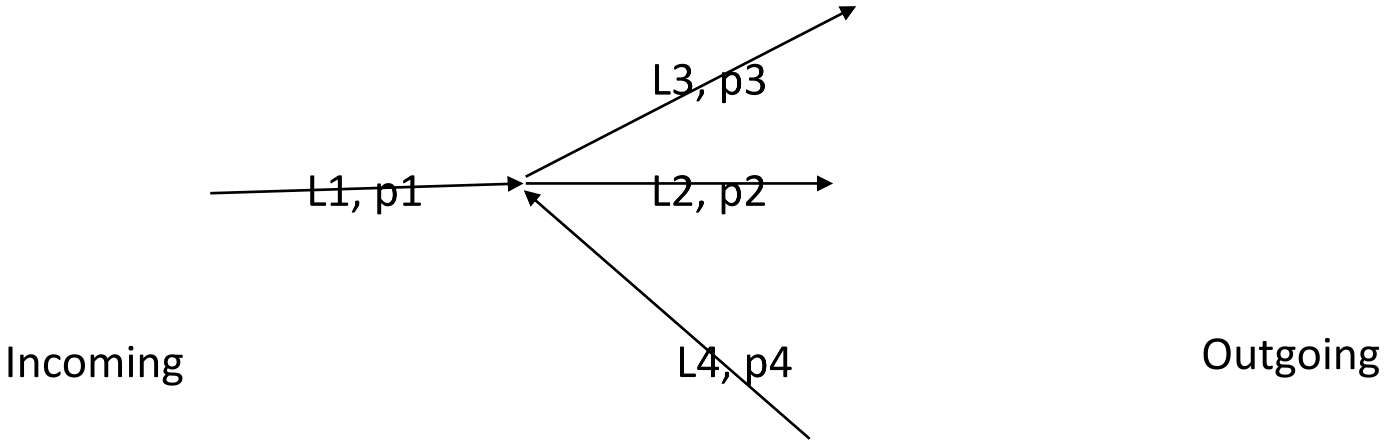
Thomas precession is a purely kinematic result of Lorentz transformations on spin ½. No g-factor.

Fermilab results announced April 7, 2021

The new experimental world-average results announced by the Muon g-2 collaboration today are:
g-factor: 2.00233184122(82)
anomalous magnetic moment: 0.00116592061(41)

Feynman Diagrams I

External lines represent real particles. Time goes from left to right. Each line has a momentum.
Momenta at vertices are conserved (e.g. p1=p2+p3+p4)

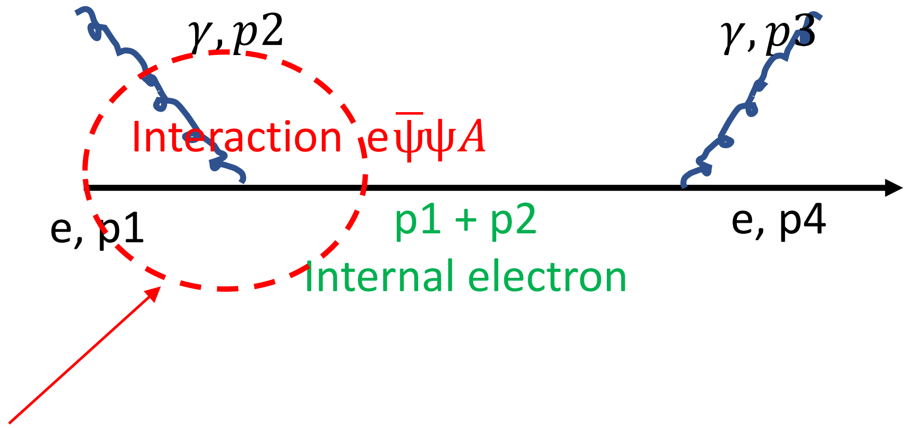


Conventions: If the direction of the arrow is left to right, it is a particle, otherwise antiparticle.

- Vertices represent interaction terms: above there is an interaction term that looks like L1 x L2 x L3 x L4.

Feynman Diagrams II

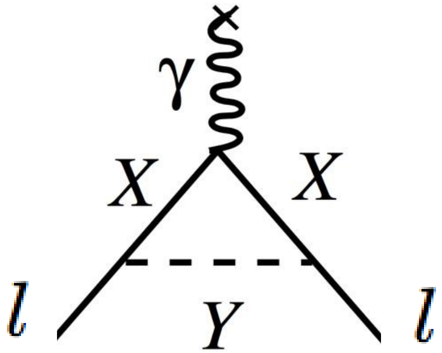
- Internal lines represent ‘virtual particles’: Shorthand for propagators or Green functions of the form $\frac{1}{p^2-m^2}$.
- The perturbation series is in “e” (see the interaction vertex below).



Represents an electron absorbing a photon and then emitting a photon. This is what happens when electron interacts with an external EM field.

The calculation involves $\frac{e^2}{(p1+p2)^2-m^2}$. There are 2 vertices so a factor of e^2.

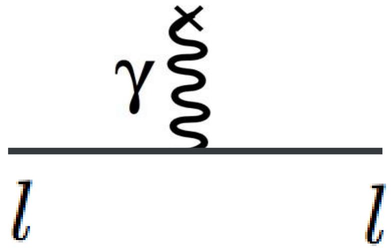
This interaction vertex can be expanded to represent higher order corrections



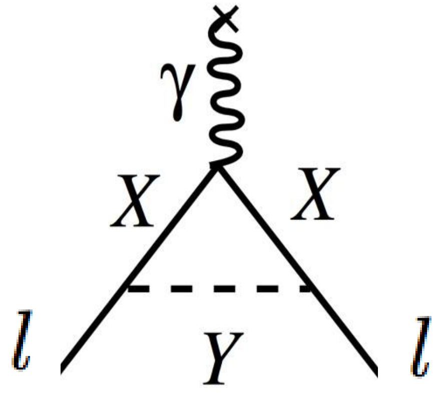
- Loops represent integrals over momentum space.

Magnetic Moment & Virtual Loops

- For a pure Dirac spin-½ charged fermion, g is exactly 2

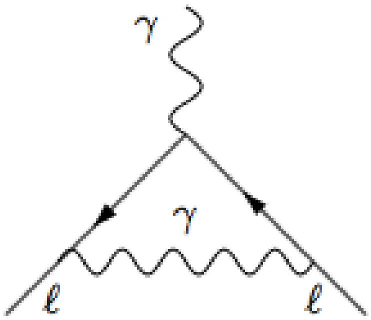


- Interactions between the fermion and virtual loops change the value of g
- X & Y particles could be standard model or new physics:



Schwinger Correction

- The most simple correction is 1st order QED, calculated by Schwinger in 1948:



$$g = 2\left(1 + \frac{\alpha}{2\pi}\right) \approx 2.00232$$

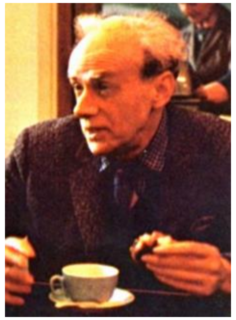
$$\alpha = \frac{e^2}{4\pi}$$

- Resolved the discrepancy in g_e as measured by Kusch-Foley in 1947

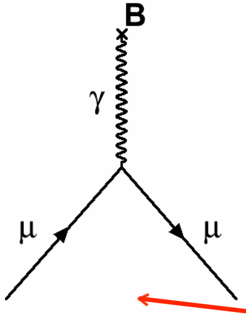
- This correction would be the same for the muon.

Standard Model Components of muon g-2 (from 2019)

Dirac

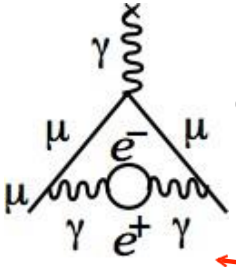


Charged, spin ½ particle

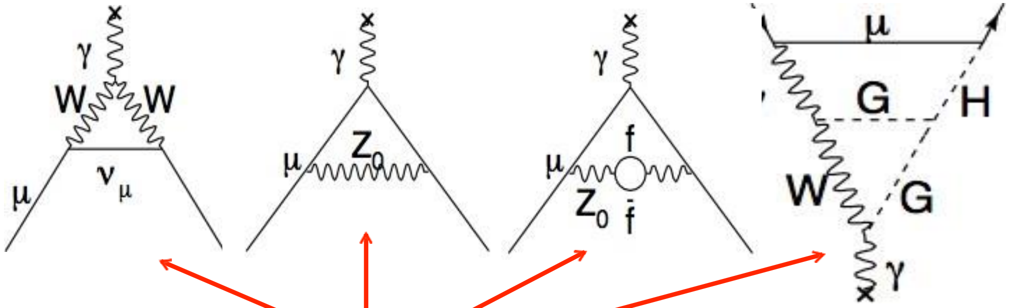


Kinoshita

Up to 10th Order QED



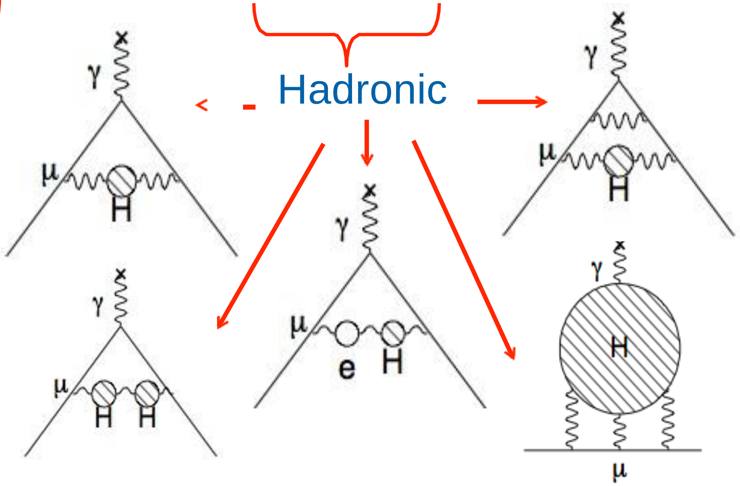
+12671 diagrams



Electroweak

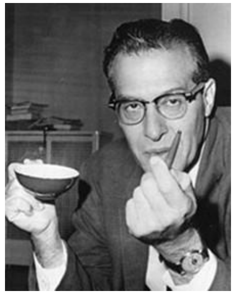
$$g_\mu = 2.00233183636(86)$$

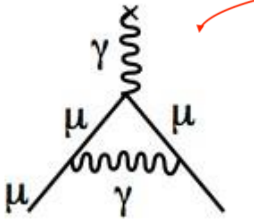
Hadronic



SM uncertainty

Schwinger





$$\frac{\alpha}{2\pi} = 0.00232$$

1st Order QED

a_μ Theoretical Status

Contribution	Value ($\times 10^{-11}$)	Reference
QED	116 584 718.95 ± 0.08	PRL **109** 111808 (2012)
EW	153.6 ± 1.0	PRD **88** 053005 (2013)
HVP (LO)	**6931 ± 34**	EPJ C **77** 827 (2017)
HVP (LO)	**6933 ± 25**	PRD **97** 114025 (2018)

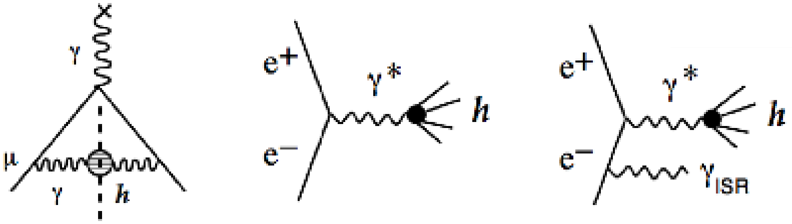
HVP (LO): Lowest-Order Hadronic Vacuum Polarization

- **Critical input** from e^+e^- colliders (data from SND, CMD3, BaBar, KLOE, Belle, BESIII), $\delta a_\mu^{HVP} \sim 0.5\%$; extensive physics program in place to reduce δa_μ^{HVP} to ~ 0.3% in coming years

- **Progress on the lattice**: Calculations at physical π mass; goal: $\delta a_\mu^{HVP} \sim$ 1—2% in a few years (cross-check with e^+e^- data)

$$a_\mu^{\mathrm{had;LO}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{m_\pi^2}^{\infty} \frac{ds}{s^2} K(s) R(s)$$

$$R \equiv \frac{\sigma_{\mathrm{tot}}(e^+e^- \to \mathrm{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



Anomalous magnetic moment
Comparisons between theory and experiment

Fermilab (experiment) 2021:	0.00116592061(41)
Muon g-2 Theory Initiative (dispersion) 2020:	0.00116591810(43)
Borsanyi et al. (BMW) (lattice) 2021:	0.00116591956(56)
Gottlieb et al. (ETM) (lattice) 2020:	0.00116591869(150)

Error bars above are 1σ (difference between quantities has 1σ = sqrt(err1^2+err2^2))

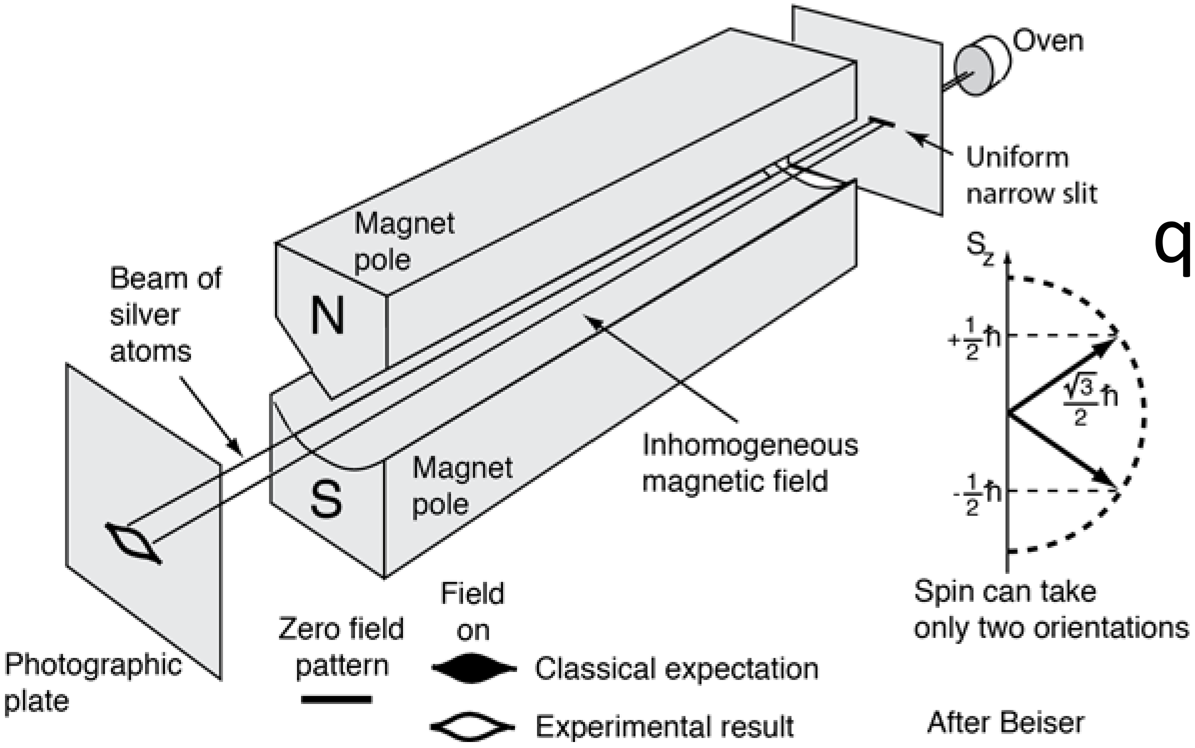
• Fermilab-dispersion:	4.2σ	cumulative probability 0.00003
• Fermilab-BMW:	1.5σ	cumulative probability 0.13
• Fermilab-ETM:	1.8σ	cumulative probability 0.07
• BMW-dispersion:	2.1σ	cumulative probability 0.04

- A difference of 5σ is considered to be a **DISCOVERY**. This is the reason for the Fermilab headline.
- Technically, since the experiment differs from the dispersion theory result by less than 5σ we call it **TENSION**.
- The BMS result is a brand-new lattice result, and BMS claims theory agrees with experiment.
- The theory community is a bit skeptical of the BMS error bars.
- More work is needed. Clearly, Fermilab wants the tax-payers to be excited!

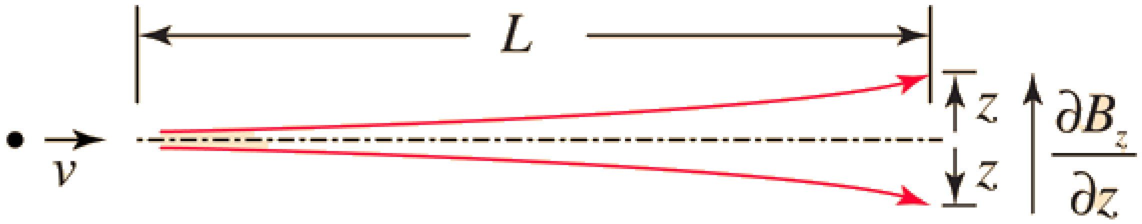
Appendix

Stern-Gerlach experiment

The earliest detection of quantization of orbital angular momentum



$$U = -\mu \cdot B = -\mu_B \frac{g}{2} B_z = \pm \mu_B B_z$$



$$F_z = -\frac{\partial U}{\partial z} = \pm \mu_B \frac{\partial B_z}{\partial z}$$

$$z = \frac{1}{2} a t^2 = \frac{1}{2} \frac{F}{m} \left[\frac{L}{v} \right]^2 = \pm \frac{\mu_B L^2}{4KE} \frac{\partial B_z}{\partial z}$$