Quark exercises

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Some recent experiments claim to have produced tetra-quark particles – mesons built out of 4 quarks. One of the latest results was submitted for publication in March 2021 by a collaboration from the Large Hadron Collider at CERN. The authors speculate that this particle has composition "$\bar{c}cu\bar{s}$". This notation is read as "anti-charm-quark, charm-quark, up-quark, anti-strange-quark".

Recall articles have the same mass but opposite quantum numbers (e.g. strangeness, charge, etc) from their anti-particles

Recall that the strange quark has a strangeness of -1 and a charge of -1/3. The up quark has a strangeness of 0 and a charge of +2/3.

PROBLEM 1: What is the expected strangeness and approximate mass of this newly discovered tetra-quark meson?

Assumptions

- The Hamiltonian of the tetra-quark system is

$$H = m_u + m_s + m_c + m_c + V(\mathbf{x_1}, \mathbf{x_2}, \mathbf{x_3}, \mathbf{x_4}) + A\frac{\sum s_i \cdot s_j}{m_i m_j} \tag{1}$$

  where V is some potential energy dependent on the 4 positions of the quarks (and probably linear in the distance between quarks) and A is a spin-coupling coefficient.[1]

[1] The form of the Hamiltonian is actually far more complicated but for the purposes of this exercise, assume that all potential energy and spin interactions contribute negligible energy to the lowest-energy eigenfunctions.

- The most significant energy contribution of the Hamiltonian comes from the mass-energies of the quarks so you can ignore contributions of the potential V and the term proportional to A. [2]

- Assume, based on fitting masses of more conventional particles, that $m_u = 300\text{MeV}$, $m_s = 500\text{MeV}$, $m_c = 1500\text{MeV}$.

PROBLEM 2 – HARDER: The new particle turns out to have spin 1. What other possible spins could it have had?

Hints

- Recall that two quarks, each of spin 1/2, can combine to form degenerate-mass states of spin 0 and spin 1. In group language, we write

$$\frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1 \tag{2}$$

where $\frac{1}{2}$ denotes the 2-dimensional (spin 1/2) irreducible representation, 0 denotes the 1-dimensional (spin 0) irreducible representation, and 1 denotes the 3-dimensional (spin 1, also known as vector) irreducible representation.

- Tensor product decompositions follow the associativity and distribution principles – in particular

$$(A \otimes B \otimes C \otimes D) = (A \otimes B) \otimes (C \otimes D) \tag{3}$$

and

$$(A \oplus B) \otimes (C \oplus D) = (A \otimes C) \oplus (A \times D) \oplus (B \otimes C) \oplus (B \otimes D) \tag{4}$$

- A tensor of representation N, when tensor-producted with a scalar, is a tensor of representation N. Symbolically,

$$A \otimes 0 = A \tag{5}$$

- Reminder of tensor product of vectors.

$$1 \otimes 1 = 0 \oplus 1 \oplus 2 \tag{6}$$

where, as mentioned above, spin 1 has dimension 3 and spin 2 has dimension 5.

[2] In the future, excited states – particles with higher masses – might be found but assume the first ones found are the lightest.