Towards SU(3)

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1 What we learned from rotational symmetry of the Coulomb potential

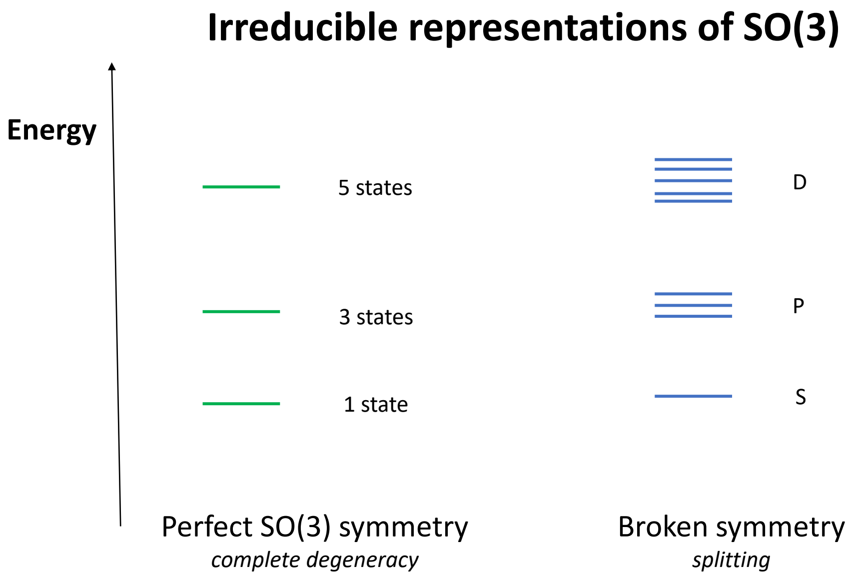
- Members of the Hilbert space are represented by complex functions.
- The equations of motion are (differential) equations acting on members of the Hilbert space.
- The equations of motion (Schrodinger equation) are invariant under rotations of the coordinates.
- Members of the Hilbert space (functions) transform into one another under rotations. For example, the three spherical harmonics Y_1^m transform into one another when we transform the θ and ϕ coordinates to represent a rotation.

$$
\begin{aligned}
Y_1^{-1} &= \sqrt{\frac{3}{8\pi}} e^{-i\phi} \sin\theta \\
Y_1^{0} &= \sqrt{\frac{3}{4\pi}} \cos\theta \\
Y_1^{1} &= -\sqrt{\frac{3}{8\pi}} e^{i\phi} \sin\theta .
\end{aligned}
\tag{1}
$$

  For each rotation, the transformation can be represented by a 3×3 matrix called $\mathbf{D(R)}$. This example is called *a 3-dimensional representation of the rotations* and the Y_i^m are called *the basis elements of the representation.*

- When we have a representation of the rotations, with the additional property that for every basis element b_α can be transformed to another basis element b_β by some rotation, then we call this *an irreducible representation.*

- Rotational symmetry implies that the Hamiltonian commutes with the n-dimensional rotation representation, $[\mathbf{H}, \mathbf{D}(\mathbf{R})] = 0$. When the representation is irreducible, the Hamiltonian has n eigen-vectors (or eigenfunctions) with the same eigenvalue E. That is, there are n different vectors ψ_m so that $H\psi_m = E\psi_m$. We say that E *is an eigenvalue with degeneracy* n.
- The rotation group has irreducible representations of dimensionalities 1, 3, 5, ... (all the odd numbers).



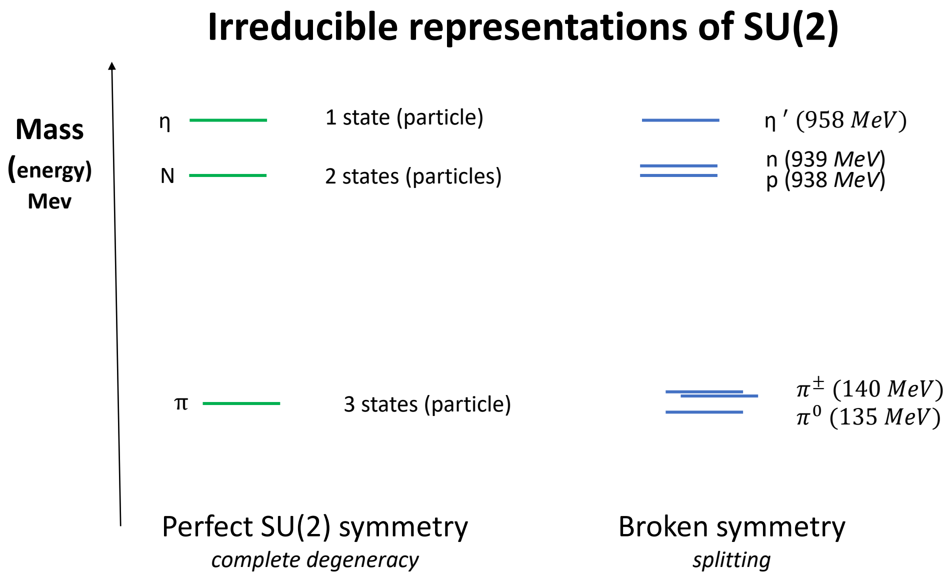
2 Irreducible representations of SU(2) – Isospin

- The group SU(2) is defined as the Lie group of 2×2 unitary matrices with determinant 1, such as

$$\mathbf{u} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} . \tag{2}$$

- We've already studied this group since it is a natural extension of the rotation group. It has the same irreducible representations (irreps) as the rotation group and in addition has even-numbered-dimension irreps also known as spinor representations. In particular, the lowest-dimension irreps are of dimension 1, 2 and 3.

- These patterns were seen in the earliest known baryons.



- A new symmetry was proposed. **IT WAS SU(2) AND WAS CALLED ISOSPIN SYMMETRY. It operated on baryon states and transformed one kind of baryon into another.**

- Life is more complicated than simply measuring energies or masses. In chemistry, reactions occur. In particle physics, scattering occurs. In both cases, symmetries imply conservation laws.

  For example, if you scatter (collide) 2 baryons, their isospin is preserved. If we were talking about rotational symmetry, we'd say something like "the z component of angular momentum must be preserved". For baryons, we say "the z component of isospin must be preserved". **That happens (approximately).** We refer to some of these conserved quantities as *quantum numbers*.

3 Irreducible representations of SU(3)

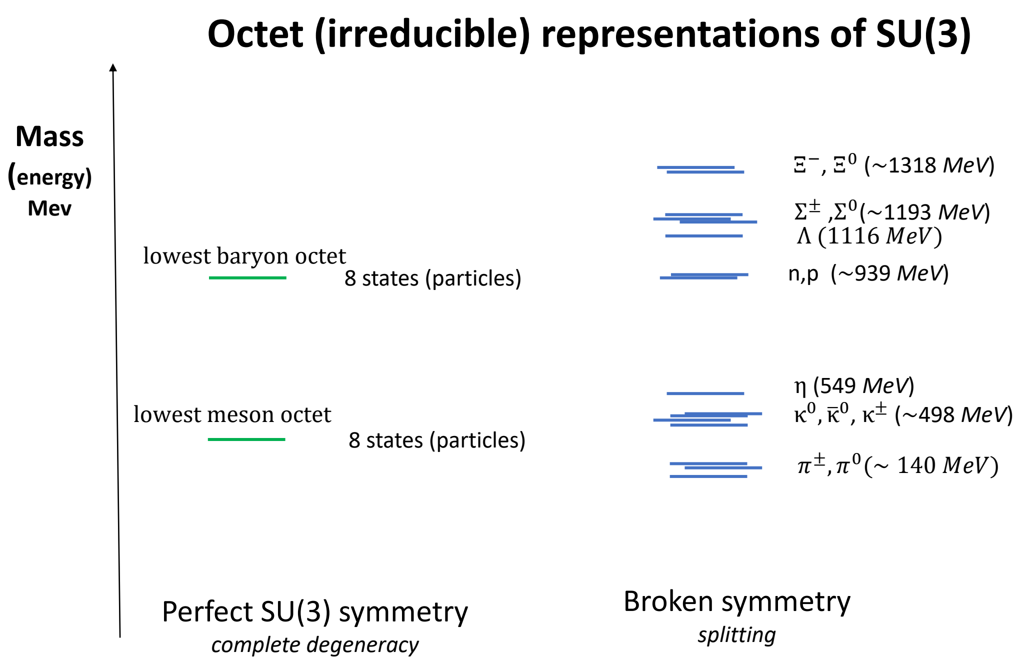
- The group SU(3) is defined as the Lie group of 3×3 unitary matrices with determinant 1. So, for example, consider the matrices $\mathbf{U}_1$ and $\mathbf{U}_2$

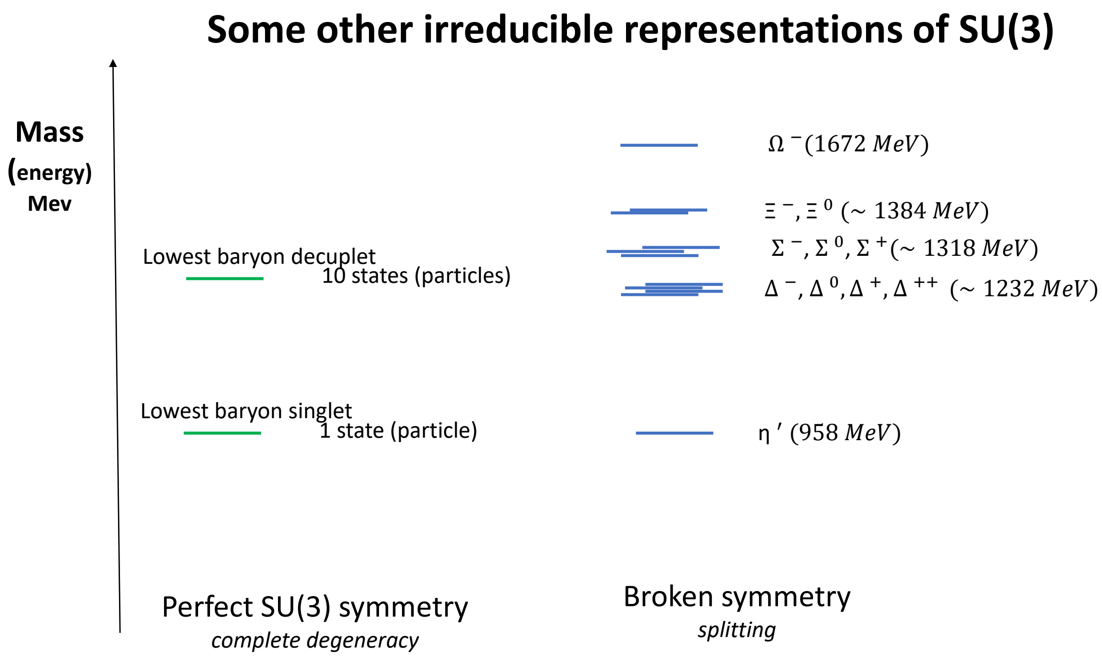
defined by

$$\mathbf{U}_1 = \begin{pmatrix} 0 & i & 0 \\ i & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} , \; \mathbf{U}_2 = \begin{pmatrix} 0 & 0 & i \\ 0 & i & 0 \\ 1 & 0 & 0 \end{pmatrix} \tag{3}$$

These are both unitary with determinant 1. Their product is also unitary with determinant 1 and therefore an element of SU(3).

- The dimensions of the lowest-order irreducible representations are 1, 3, 6, 8, and 10. These patterns were seen as more baryons were discovered.





- The groups of baryons are characterized by a variety of quantum numbers such as isospin, charge, overall spin etc. Furthermore, the isospin groupings are contained within them.

- A new symmetry was proposed, which contained the isospin symmetry SU(2). **IT WAS SU(3) AND IS NOW CALLED FLAVOR-SU(3). When Murray Gell-Mann proposed the symmetry, the Ω particle hadn't yet been found. He predicted it and won the Nobel Prize for his work.**

- QUESTION: Where are the 3-dimensional and 6-dimensional representations? ANSWER: These have never been seen. The 3-D representation particles are called ***quarks***.