Thomson Chapter 1.1 – The Standard Model

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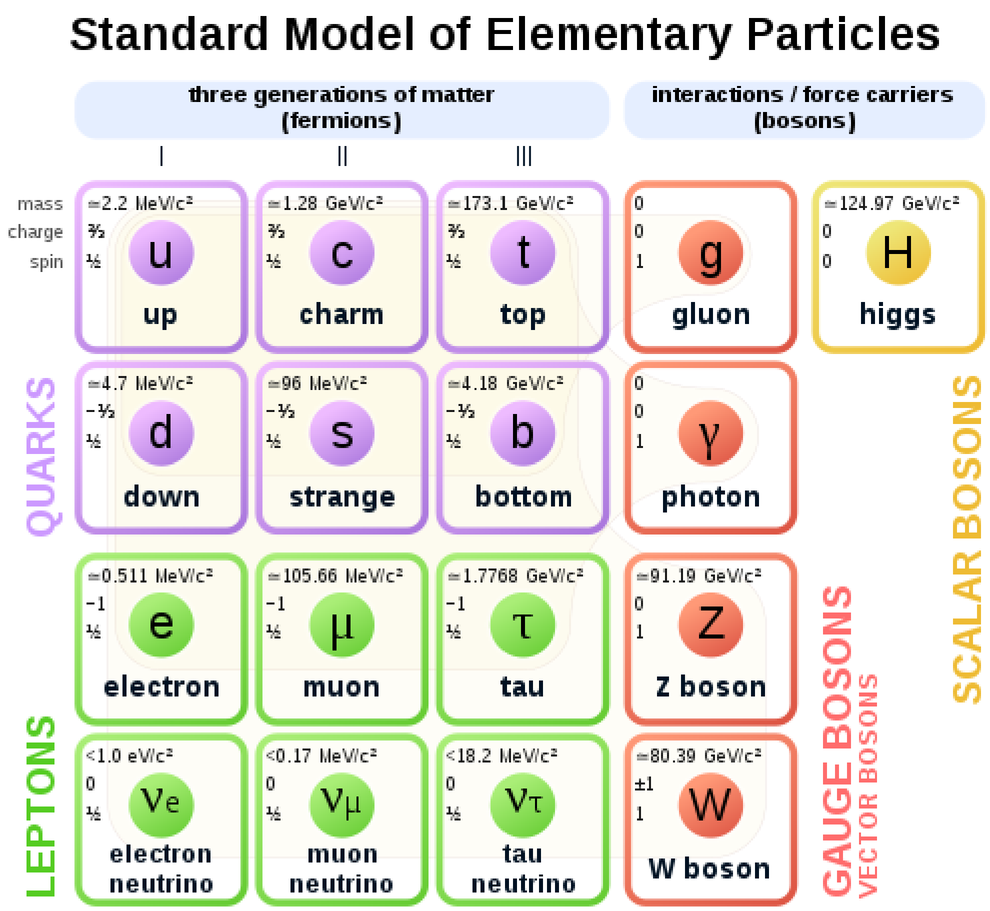


Figure 1: Standard Model Elementary Particles
(diagam from Wiki page https://en.wikipedia.org/wiki/Standard_Model)

1 Particles, forces and fields

- Thomson follows the common convention of treating particles and forces as separate entities. For example, according to Thomson, electrons are particles whereas electromagnetism is a force.

- However, this separation obscures the critical role of quantum fields. It also tends to obscure the essential intuition of Newton – namely, that force-at-a-distance must somehow be caused by the action of some concrete 'thing' going from one particle to the other.

- In a quantum field theory, each field is both associated with a particle, and also is associated with a force. The 'force' aspect of the field is associated with the Euler-Lagrange equations for the field. The 'particle' aspect of the field arises from the fact that the field is a linear operator. What it operates on are the states of a Hilbert space – states that represent particles.

- For example, the electromagnetic field operates on photon states. The way in which the electromagnetic field interacts with itself and with other fields is dictated by Euler-Lagrange equations and indirectly this leads to forces.

- Physicists use an illustrative language to say all this: we say 'the electromagnetic force is mediated by photons', and we draw pictures that look as though photons travel between the charged particles that are attracted/repelled to/from one another. This then fits neatly into Newton's intuition although the mathematical details are probably very different from what he imagined.

- To see why we need to be careful, consider the following: Weak forces are associated with the vector bosons $W^{\pm}$ and Z. So we think of the weak force as 'carried' by those vector bosons. But we can also regard these vector bosons as elementary particles that interact with one another by 'exchanging' other vector bosons. Or, even more provocative, one can regard photons as particles which scatter from one another on account of the force 'carried' by electrons.

- In short, every force is associated with a field, which is also associated with a particle and vice versa. None of this really contradicts what Thomson has said, nor the conventional way of talking about forces and particles. However, in my opinion, the nomenclature 'elementary particles' should embody not only of the leptons and quarks, but also the photon, gluons, W and Z bosons (and gravitons). Likewise the nomenclature 'force' should embody not only the familiar forces 'carried by' photons, gluons etc. but also the forces carried by quarks, leptons etc.

2 Vertices and Feynman diagrams

2.1 Mnemonics for the Feynman diagram on page 8 (Fig. 1.5)

We see two vertices (in this case, strong-coupling vertices) joined together. This diagram is built using a mnemonic for interpreting the meaning of vertices, free lines and joined lines. The diagram is then used (see next subsection) *precisely* as a schematic representing a particular mathematical expression which contributes to scattering or decay probabilities. **Here are the mnemonics that Thomson implicitly uses in this chapter.**

- The figure is an example of a Feynman diagram.

- By convention (not universal), free lines on the left are interpreted as incoming particles (e.g. a single particle decays, or two particles collide) and free lines on the right are interpreted as outgoing particles (the end-result of a decay or collision). In this sense, **and only in this sense** we say that time goes from left to right.

- Vertices are interpreted as in Fig. 1.4. They describe what transitions are allowed. For example, in Fig. 1.4, the electromagnetic example shows an incoming electron which emits a photon and another electron (of different momentum) or alternatively, an incoming electron which absorbs a photon and emits another electron (in Fig. 1.4, the photon is shown as neither on the left or right). So, in Fig. 1.5, the top vertex represents a transition where a fermion emits a boson and another (i.e. having a different momentum) fermion. **Alternatively the fermion absorbs a boson and emits another fermion.**

- Connected lines (i.e. joined to vertices at both ends) are interpreted as 'virtual particles' (more on this shortly). Think of them as though they are traveling from the top vertex to the bottom vertex – in our example we would say that the top vertex has emitted a boson **OR from the bottom vertex to the top vertex – in our example, we would say the fermion has absorbed a boson.** In other words, the direction of time is unspecified for connected lines. *This time-independence is one of the reasons we call the particles 'virtual'.*

- Remember that the diagram is a mnemonic. In particular, only the free lines are directly representative of experiments. Everything else in the diagram is un-measurable. On the other hand, these diagrams correspond rather nicely to Newton's intuition that the forces of nature come from the exchanges of some concrete 'things' that enable the concept of action-at-a-distance.
- Feynman diagrams often label the lines with momenta. Again, only the free lines have momenta which are measurable.
  - A rule (not mentioned before) is that at every vertex, the total 4-momenta must be conserved in the sense that the incoming total must be equal to the outgoing total.
  - For connected lines, there is an ambiguity about incoming and outgoing. Pick one and calculations will turn out not to care.
  - Since free lines represent measurable particles, their momenta must obey the dispersion relation $E^2 = \mathbf{p}^2 + m^2$, or in 4-momenta notation we write p_0 instead of E. We say that *the particles are on their mass-shells.*
  - It turns out that we can't simultaneously conserve momenta at the vertices, and demand that the connected lines represent particles on their mass-shells. For example, in Fig. 1.4, the vertices don't actually describe physically measurable processes. If you demand conservation of momentum, it turns out this can't happen if all particles obey the dispersion relations. In general, *the virtual particles have momenta that do not obey the above dispersion relations.* This is yet another way in which the connected lines don't correspond to anything experimentally measurable and are therefore called 'virtual'.

2.2 Mathematical interpretation of the Feynman diagram on page 8 (Fig 1.5)

2.2.1 Vertices on page 7 (Fig. 1.4)

The pictures are schematic representations of interaction terms in the Lagrangian and illustrate what processes (interactions) are possible. The complete set of Lagrangian terms is huge. What follows are examples of electromagnetic, strong and weak interaction terms.

- Electromagnetism:

$$\begin{aligned}\mathcal{L}_{EM} &= -e\bar{\psi}^e \not{A}\psi^e \\ &\equiv -e \sum_{i,j,k,\mu} \psi_i^{e\dagger}\gamma_{ij}^0\gamma_{jk}^\mu A_\mu \psi_k^e + ...\end{aligned} \tag{1}$$

e is the electromagnetic coupling constant, also known as the electric charge, ψ^e is the Dirac spinor of the electron field, A^μ is the electromagnetic four-potential field operator and indices i, j, k are spinor indices (ranging from 1 to 4).

- Strong interaction:

$$\mathcal{L}_{QCD} = g_s \bar{\psi}_m^q \not{G}^a T_{mn}^a \psi_n^q + ... \tag{2}$$

g_s is the strong coupling constant, ψ^q is the Dirac spinor of the quark field, G_μ^a is the octet of gluon-field 4-vectors (the index a ranges from 1 to 8) and T^a are the 3x3 Gell-Mann matrices (generators of SU(3)). The indices m, n are known as color indices. Numerically, they range from 1 to 3, but are often known as the colors {r,g,b}. Notice that we have, in this vertex, selected one particular flavor of quark (for example 'down' or 'strange').

We see that the vertex involves Dirac spinors with the same flavor. That is, the QCD interaction doesn't change flavor.

- Charged weak interaction:

$$\mathcal{L}_W = \frac{g_W}{\sqrt{2}} \bar{\psi}_L^e (\not{W}^-) \psi_L^{\nu_e} + ... \tag{3}$$

The coupling constant is g_W (or, if you prefer, $g_W/\sqrt{2}$). The field W_-^μ is known as the negatively charged weak vector boson (although people usually just call it the W_-). The fermion fields have a subscript L to denote that they are multiplied by the projection matrix $P_L = \frac{1}{2}(\mathrm{I} - i\gamma^0\gamma^1\gamma^2\gamma^3)$. Then ψ_L^e is the Dirac spinor of the 'left-handed electron' field and $\psi_L^{\nu_e}$ is the Dirac spinor of the 'left-handed electron-neutrino' field.

- Neutral weak interaction:

$$\mathcal{L}_Z = g_Z \bar{\psi}_L^{\nu_e} \not{Z} \psi_L^{\nu_e} + ... \tag{4}$$

The coupling constant is g_Z. The field Z^μ is known as the Z boson.

2.2.2 Perturbation Theory

The mathematical interpretation of Feynman diagrams is in the context of the calculation of either decay amplitudes or scattering amplitudes. If an amplitude is denoted A, then the probability is denoted $|A|^2$. Feynman diagrams are schematics for calculations of amplitudes.

A decay amplitude (per unit time) is the amplitude that a single particle decays into a particular set of outgoing particles. Also of interest is the cumulative decay amplitude for the particle to decay into anything. This quantity is inversely proportional to the particle's lifetime.

A scattering amplitude is the amplitude that two colliding particles produce a particular set of outgoing particles.

- Amplitudes are derived from path integrals by computing moments of the form

$$G^{i_1 \dots i_n}(y_1, \dots, y_n) \equiv \int \mathcal{D}\phi^i e^{i(\int d^4x \mathcal{L}(\phi^i))(x)+i\epsilon)} \prod_{j=1}^{n} \phi^{i_j}(y_j). \quad (5)$$

  In this expression, the Lagrangian is a function of fields shown generically as ϕ but which can represent, for example A^μ or ψ^{ν_e}.

- If the Lagrangian were quadratic in the fields, then these moments could be computed exactly, in the same way that moments of a Gaussian distribution can be computed exactly.
- This enables us to compute perturbatively. Rewrite the Lagrangian generically as

$$\mathcal{L} = \mathcal{L}_0 + \lambda \mathcal{L}_I \quad (6)$$

  where $\mathcal{L}_0$ is the part of the Lagrangian which is quadratic (this part is called the free Lagrangian) and $\lambda \mathcal{L}_I$ is the rest, and is called the interaction term. Here λ is a coupling constant which, for the purposes of perturbation theory, must be small. **Example interaction terms, representing the schematics of Fig. 1.4, are given in the previous subsection**.

- Then we can expand the exponential in the path integral as

$$e^{i\int d^4x(\mathcal{L}_0+\lambda\mathcal{L}_I+i\epsilon)} = e^{i\int d^4x(\mathcal{L}_0+i\epsilon)} e^{i\lambda\int d^4x\mathcal{L}_I}$$
$$= e^{i\int d^4x(\mathcal{L}_0+i\epsilon)} \left(1 + i\lambda \int d^4y_1 \mathcal{L}_I + i^2 \frac{\lambda^2}{2!} \int d^4y_1 \mathcal{L}_I \int d^4y_2 \mathcal{L}_I + \dots\right) \quad (7)$$

– Now, when we compute the path-integral, the exponential term is quadratic, and the interaction piece has become a series contributing to the moments – all of which are calculable.
– Consider the term $i^2 \frac{\lambda^2}{2!} \int d^4y_1 \mathcal{L}_I \int d^4y_2 \mathcal{L}_I$ where, for example, we take $\lambda\mathcal{L}_I$ to be the electromagnetic term from the last subsection. This becomes[1]

$$-e^2 \int d^4y_1 d^4y_2 \left(\bar{\psi}^e \not{A} \psi^e\right)(y_1) \left(\bar{\psi}^e \not{A} \psi^e\right)(y_2) \tag{8}$$

– **This represents the schematic of Fig 1.5**. The vertex appears twice. It turns out that we don't have to path-integrate the fields which appear as free particles (i.e., altogether 4 fermion fields). The moment-integral only has to do with the two appearances of A. This is the meaning of the connected line. The moment-integral corresponding to the connected line is known as the propagator and has the generic form $\frac{\text{stuff}}{p^2 - m^2 + i\epsilon}$. The momentum appearing in this expression is the momentum mentioned in the previous section and which, as mentioned in the previous section does **not** have the property that $p^2 = m^2$.[2]

The most important take-away for this chapter, is that the amplitude in Fig 1.5 is proportional to e^2 and for other Feynman diagrams, is proportional to the product of the coupling constants appearing at the vertices of those diagrams.

2.3 Problem 1.1 on page 28 – solutions

Most rules can be found on page 7 in Fig. 1.4.

- Part (a): YES. This diagram is the electromagnetism diagram of Fig. 1.4.
- Part (b): NO. Photons (γ) only join vertices with charged particles. Neutrinos (ν_e) aren't charged (see Table 1.1).
- Part (c): NO. Photons preserve charge. This diagram shows a negatively charged electron becoming a positively charge positron. (See page 10 – the sentences just before the beginning of 1.1.6).

[1]For sticklers, please note that we haven't ever said what we mean by path integrals involving fermions. Technically, these fields are regarded as members of a Grassman algebra, and integration is defined rather differently than it would be for regular complex-valued fields. However, miraculously, the formalism works with few modifications.

[2]Since gluons are massless, this particular example would have $m^2 = 0$.

- Part(d): YES. This diagram is shown for weak interactions in Fig. 1.4.
- Part(e): NO. The photon doesn't change flavor. The electron and muon are different flavors.
- Part(f): YES. This diagram is shown for weak interactions in Fig. 1.4.
- Part (g): NO. The Z-boson doesn't change flavor. The electron and tau are different flavors.
- Part (h): YES. (At least, I can't think of anything prohibiting his vertex.) The W-boson is allowed to change flavor (and generation).
- Part (i): NO. The gluon only couples to quarks, and not electrons.
- Part (j): YES. The bottom particle is a quark, which the gluon can couple to.
- Part (k): NO. Gluons don't change flavor. Down and strange quarks are different flavors.
- Part (l): NO. Photons only couple to charged particles. Photons aren't charged.
- Part (m): NO. The W always changes flavor. The two quarks in this diagram are both the same flavor (up).
- Part (n): YES. The W can change flavor. Actually, it also changes charge. The up and down quarks have different flavors and different charges.
- Part (o): YES. Just as for part(n). The down and top quark are different flavors and different charges.
- Part (p): NO. There is no such vertex coupling photons to charged particles. By the way, nothing prohibits vertices with 4 legs, but we haven't encountered that kind of thing yet.

3 Why do we think there are only 3 generations of fundamental fermions?

Thomson, in Table 1.1 on page 3, enumerates the fundamental fermions. For example, we see that there are two particles that are similar, in every respect except mass, to the electron. Those are the muon and the tau. The tau was only discovered in the past few decades, but the muon was found in 1936, prompting the famous experimental physics Isidor Rabi to say "who ordered that?" Some natural questions are "Why are there three generations? Why not one generation? Why not 5?" Since successive generations are increasingly massive, one natural possibility is that we've simply been unable to generate enough collider energies to produce particles more massive than those of the third generation. Here's some of the current thinking.

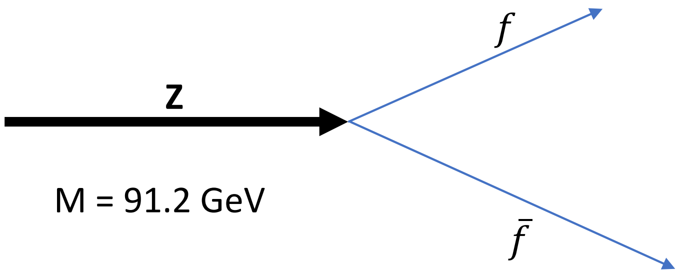
3.1 General considerations

- We assume that if there were a 4th generation (for example), that its members would behave more or less the way members behave in the other generations. That is, those fermions would couple to gauge bosons with similar coupling strengths found in the other 2 generations. So the fourth-generation quarks would couple to gluons, the 4th generation charged lepton would have charges of + and - (as opposed to charges of +2 and -2 or +3 and -3, etc.).

- We also assume that if there were a 4th generation, the 4th-generation neutrinos would be exceptionally light, just like the lower-generation neutrinos. This turns out to be a crucial assumption.

- Notice that the other leptons (electron, muon, tau) are all relatively light compared to the top quark and, in particular, the Z-boson. However, in looking at the ratio of masses between the top quark and the charm quark (the +2/3-charge members of the second and third generation) we see that it's possible for generational masses to have ratios of a couple of orders of magnitude. So it might not be unreasonable to think that a 4th generation version of the electron would have a mass of over 100 GeV.

3.2 Decays of the Z-boson



This Feynman diagram shows a Z-boson decaying into a fermion plus anti-fermion pair. The decay can happen provided that the fermion and anti-fermion together have rest masses that total less than 91.2 GeV. That way, energy can be conserved (the fermion and anti-fermion each would have some kinetic energy) and momentum could be conserved by having the two product particles go off in opposite directions.

- Lifetimes are indirectly given in terms of a quantity called Γ. Γ_Z is a measure of the total lifetime of the Z boson. This can be directly measured in scattering experiments. $\Gamma_Z = 2495$ **MeV.**

- It's also possible to directly measure the decay probability into hadrons (which are produced by quarks) and charged leptons. $\Gamma_{\mathbf{quark}}+\Gamma_{\mathbf{charged\ leptons}} =$ 1997 **MeV.**

- The Z-boson also decays into neutrino/anti-neutrino pairs. These interact so weakly that they pass through detectors so that their decay probabilities can't be measured. However, theory predicts fairly precisely what they contribute. According to theory, each species of neutrino contributes the same as any other species, so for example $\mathbf{\Gamma}_{\nu_e} = 168$ **MeV, according to theory.**

- With exactly 3 generations, this prediction when added to the hadron and charged-lepton contributions gives $\mathbf{\Gamma}_{\text{Z-predicted}} =$ **2501** MeV.

- We see this is extremely close to the total measured lifetime of the Z. One more generation would give different results. Even if we were to assume that the charged particles of the 4th generation were all too heavy for Z to decay into them, the neutrino are unlikely to be too heavy and therefore should contribute. But clearly there is no such contribution.