Thomson Review of Chapters 2 and 3

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- The two primary situations studied in QFT are:
  - Decay: One particle occasionally deteriorates into other particles.
  - Scattering: Two particles collide and might transform into other particles.
- In both cases, the initial and final configurations are described as initial and final 'states'. Those initial and final states, if nonrelativistic, are described by the multiparticle Schrodinger equation for free particles. Each particle's wave packet obeys (in natural units)

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2m}\frac{\partial^2\psi}{\partial x^2} \tag{1}$$

  which is easily solved and describes 'quantum straight line motion'.

  For relativistic quantum theory, similar kinds of equations describe free particle motion. **We say they are governed by a free Hamiltonian H_0**

- We mathematically describe the above physical situations by a Hamiltonian which is written as a sum

$$H = H_0 + H_I \tag{2}$$

  where H_0 is called the 'free Hamiltonian' and H_I is called the 'interaction Hamiltonian'.

- We'll express our predictions in terms of initial and final states that are eigenstates of the free Hamiltonian. Our prediction will be obtained as "the probability density that, starting with the system in the specified initial state, it will end up after a time T in the specified final state". We divide by T to obtain the rate.

- **Fermi's Golden Rule**

$$\Gamma_{fi} = 2\pi |T_{fi}|^2 \rho(E_f) \tag{3}$$

where Γ_{fi} is the rate at which the initial state i transitions to the final state f, $T_{fi} \equiv \langle f|H_I|i\rangle$ and is called the transition matrix, and ρ is the density of final states for that initial state.

- **THE MAIN TAKEAWAY IS THAT SCATTERING AND DECAY PREDICTIONS REQUIRE TWO KEY INGREDIENTS**.

  1. **The transition matrix – which we compute in QFT with Feynman diagrams**
  2. **The density of states – which is primarily a kinematic factor that can be computed exactly independent of the Hamiltonian.**

- The density of states for energy E_f is defined by saying that the number of accessible energy states in an interval E_f to $E_f + \delta$ is $\rho(E_f)\delta$. By an accessible energy state, we mean 'a state $\langle s|$ which can be transitioned-to from the initial state described by $|E_i\rangle$'. In other words $\langle s|H_I|E_i\rangle \neq 0$.

- More generally as we proceed to particle physics, the initial state is given by the 4-momenta of each particle in the beam, and the density of final states is defined with respect to an interval around a particular configuration of final 4-momenta.

- Instead of the transition rate Γ_{fi}, experiments usually report a cross-section σ

$$\sigma = \frac{\Gamma_{fi}}{v_a + v_b} \tag{4}$$

where v_a and v_b are the speeds of the two incoming particles in a lab frame where the velocities are in opposite directions from one another. Thomson, in section 3.4, explains that the cross-section can be visualized as a measure of the effective target size that each particle presents to the other.