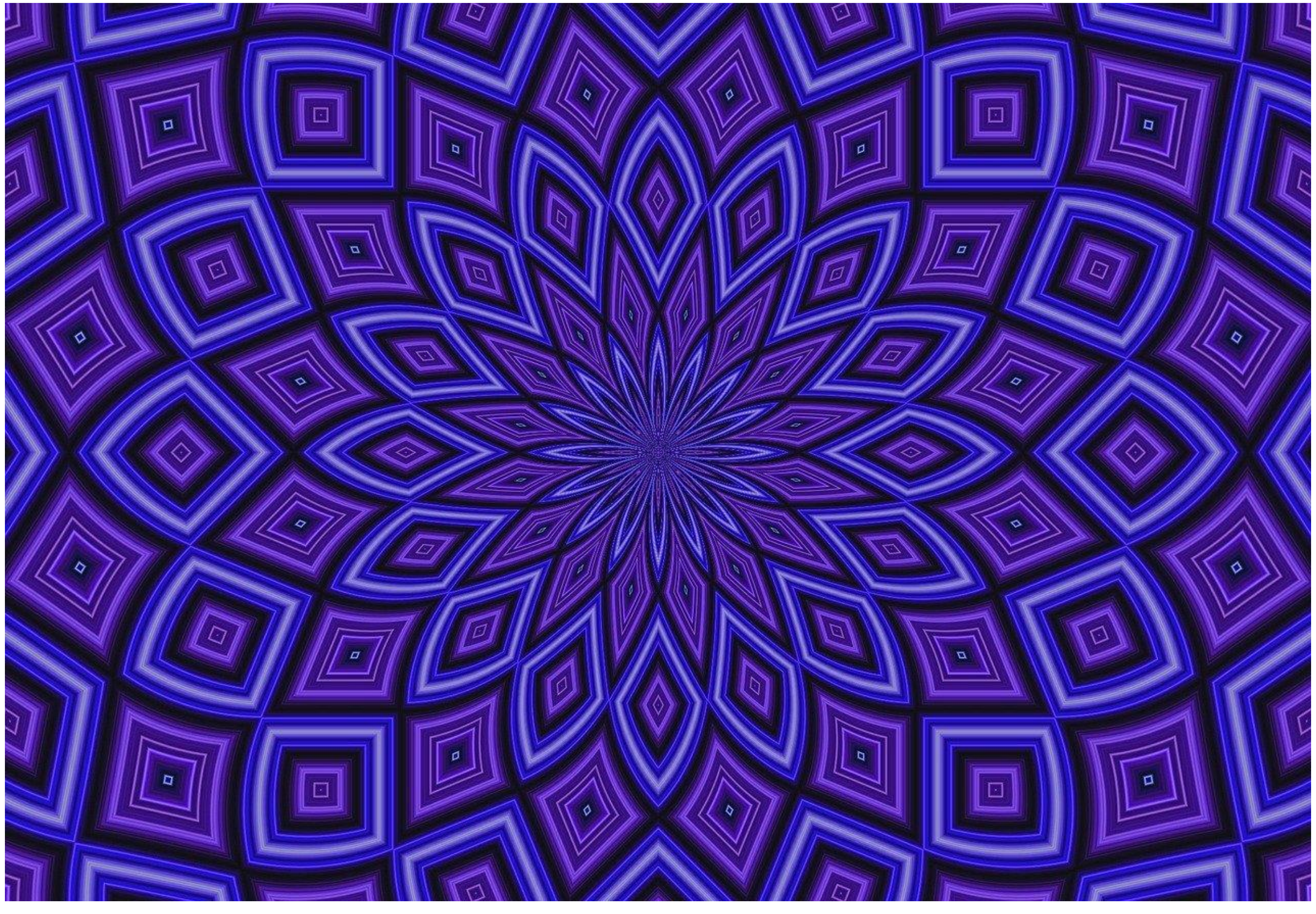
Making sense of QFT

Lecture 1: Poincaré's symmetry

by Eugene Stefanovich



QFT = quantum field theory

Good news:

Anomalous electron magnetic moment:

experiment:	0.00115965218073
QFT:	0.001159652181643

Bad news:

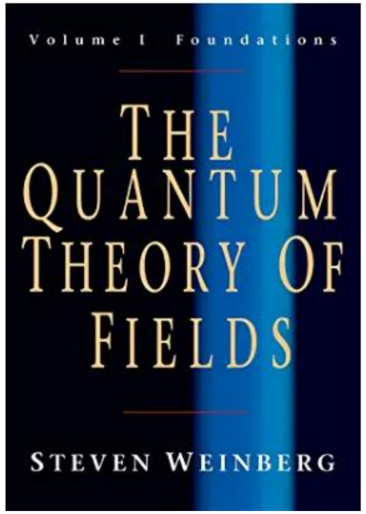
Arthur Jaffe

"...we believe that the question, “Does there exist a mathematically-complete, non-linear relativistic quantum field theory in Minkowski four-space?” remains one of the most important unresolved questions in *all of science.*" Arthur Jaffe

What is going on?

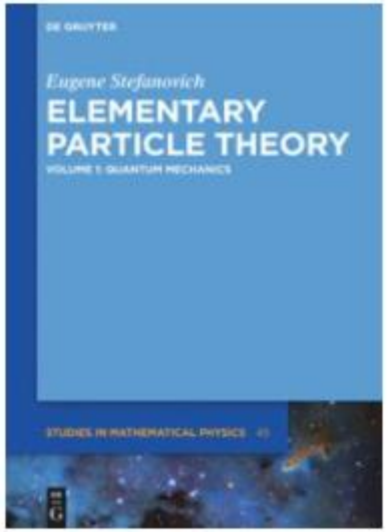
Two alternative ways to interpret Quantum Field Theory

Steven Weinberg

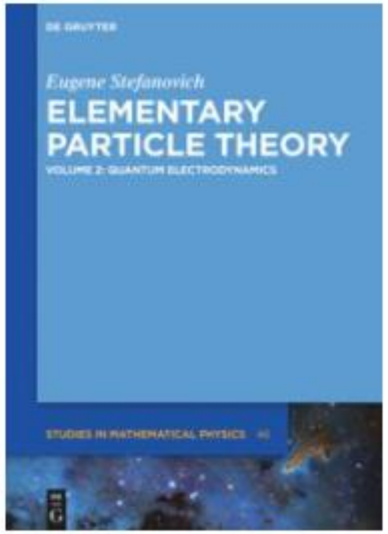


interpretation of QFT	particles	quantum fields	mathematical description
standard way	"excitations" of fields	primary ingredients of Nature	field Lagrangians and field equations
Weinberg's way	primary ingredients of Nature	useful mathematical tools	unitary representation of the Poincaré group

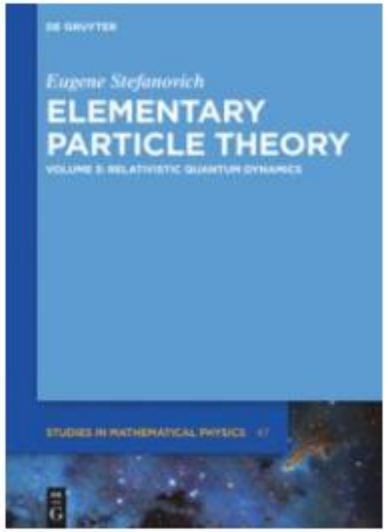
This series of lectures is based on a three-volume book titled "Elementary particle theory",



volume 1: **"Quantum mechanics"**



volume 2: **"Quantum electrodynamics"**



volume 3: **"Relativistic quantum dynamics"**

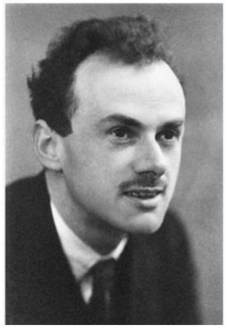
whose main message is that QFT can be reformulated as a theory of interacting particles.

Milestones of Weinberg's way (chain of ideas, in no chronological order)

Eugene Wigner

1939, Symmetry in quantum mechanics

Paul Dirac



1949, Relativistic forms of dynamics

Steven Weinberg

1964, Particle-based aproach to QFT

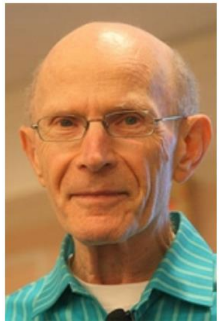
EPT, Volume 1

Richard Feynman

1949, Renormalization

EPT, Volume 2

Oscar Greenberg



Silvan Schweber

1958, Dressed particle theory

EPT, Volume 3

Outline

1. Postulates of quantum mechanics
2. Poincaré group and Poincaré Lie algebra

Postulates of quantum mechanics

- QFT is based on the same good old quantum mechanics invented by Heisenberg and Schrödinger
- QFT generalizes QM just in one sense: the number of particles in QFT is variable
- Physical observables = Hermitian operators *F* = sets of eigenvalues and eigensubspaces $F = \sum_i f_i |i\rangle \langle i|$.
- Pure states = rays of vectors $|\Psi\rangle$
- The probability of measuring value f_i in a state $|\Psi\rangle$ is

$$\omega_i = \langle\Psi|i\rangle\langle i|\Psi\rangle = |\langle\Psi|i\rangle|^2$$

- The expectation value of *F* is

$$\langle F\rangle = \langle\Psi|F|\Psi\rangle$$

Principle of relativity

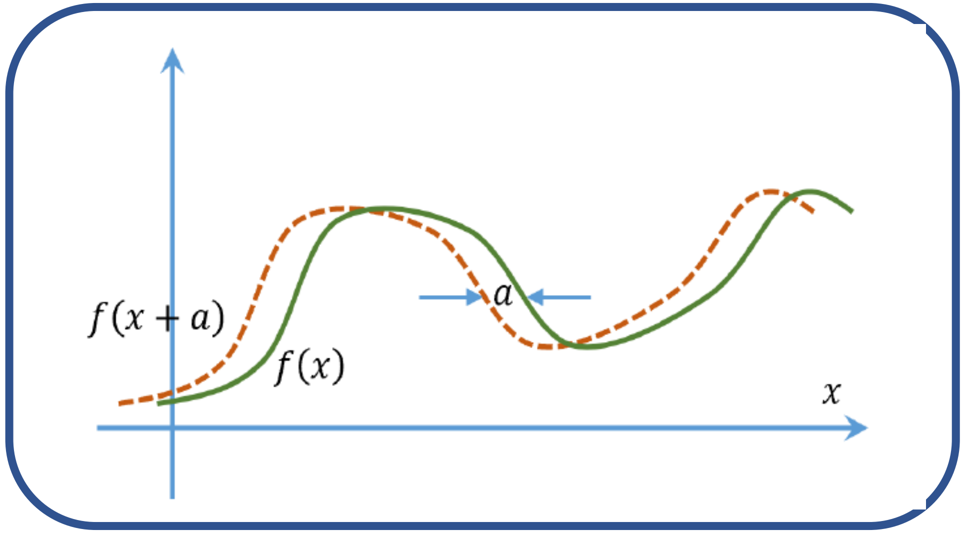
- We are not talking about the full special relativity with Minkowski space-time, Lorentz transforms, etc.
- Different inertial reference frames are equivalent.
- Inertial transformations between frames constitute the Poincaré group with 10 generators

Transformation	Generator	Group element	Parameter
space translation	**P**	$e^{\mathbf{P}\cdot\mathbf{x}}$	x = distance
rotation	**J**	$e^{\mathbf{J}\cdot\boldsymbol{\phi}}$	ϕ = angle
boost	**K**	$e^{c\mathbf{K}\cdot\boldsymbol{\theta}}$	$\theta = \tanh^{-1}(v/c)$ = rapidity
time translation	H	$e^{\mathrm{H}t}$	t = time

Example of a group generator

Group: translations of functions on the x-axis

$$f(x) \to f(x+a).$$



Using Taylor series

$$f(x+a) = f(x) + a\frac{d}{dx}f(x) + \frac{a^2}{2!}\frac{d^2}{dx^2}f(x) + \ldots$$
$$= \left(1 + a\frac{d}{dx} + \frac{a^2}{2!}\left(\frac{d}{dx}\right)^2 + \ldots\right) f(x).$$

The effect of translation is represented by the action of an exponential operator

$$f(x+a) = e^{a(d/dx)} f(x).$$

Generator of translation: "infinitesimal translation" or derivative (d/dx)

Principle of relativity: Lie algebra of Poincaré generators

$$[\mathcal{J}_i, \mathcal{P}_j]_L = \sum_{k=1}^{3} \epsilon_{ijk} \mathcal{P}_k, \quad (2.40)$$

$$[\mathcal{J}_i, \mathcal{J}_j]_L = \sum_{k=1}^{3} \epsilon_{ijk} \mathcal{J}_k, \quad (2.41)$$

$$[\mathcal{J}_i, \mathcal{K}_j]_L = \sum_{k=1}^{3} \epsilon_{ijk} \mathcal{K}_k, \quad (2.42)$$

$$[\mathcal{J}_i, \mathcal{H}]_L = 0, \quad (2.43)$$

$$[\mathcal{P}_i, \mathcal{P}_j]_L = [\mathcal{P}_i, \mathcal{H}]_L = 0, \quad (2.44)$$

$$[\mathcal{K}_i, \mathcal{K}_j]_L = -\frac{1}{c^2} \sum_{k=1}^{3} \epsilon_{ijk} \mathcal{J}_k, \quad (2.45)$$

$$[\mathcal{K}_i, \mathcal{P}_j]_L = -\frac{1}{c^2} \mathcal{H} \delta_{ij}, \quad (2.46)$$

$$[\mathcal{K}_i, \mathcal{H}]_L = -\mathcal{P}_i. \quad (2.47)$$

- The most important equations of relativistic physics
- In the non-relativistic limit $c \to \infty$ they go to Lie brackets of the Galileo group: (2.45) = (2.46) = 0

Thank you!