Thomson Chapters 5 and 6 – QED

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1 Where we've come from and where we're going

1.1 Chapter 1

- We're given the taxonomy of elementary particles, and some rules for their interactions.

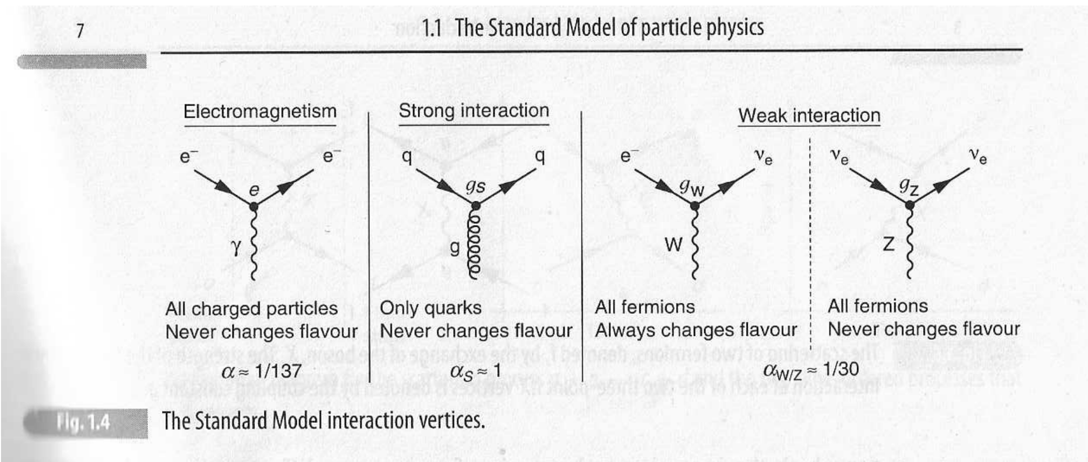
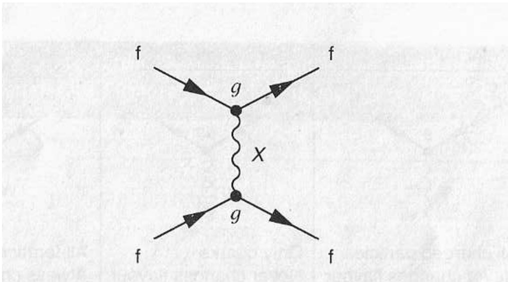


Fig. 1.4 The Standard Model interaction vertices.

- We use these vertices to compose schematics known as "Feynman diagrams". **There are schematic rules that tell us how to compute the rate of the process depicted – either a scattering process or a decay process.**
  - The end of a line (straight or squiggly) can either be connected to a vertex or it can be "free". In the above diagrams, all lines have one free end and one end connected to a vertex. We call these

"free lines". If both ends are connected, we call it a "connected line". See the squiggly line below.



- We describe the schematics with jargon:
  * A free line is called a "real particle"
  * A connected line is called a "virtual particle"
- The diagram is interpreted as time-ordered for real particles: Real particles on the left are ingoing (e.g. they are colliding with one another or decaying); real particles on the right are outgoing (the products of a collision or decay).
- Virtual particles are not time-ordered, even if they are depicted as going from left to right. We are free to regard a virtual particle as either entering a vertex, or departing a vertex.
- The real particles describe the experiment. We must specify properties such as momentum, polarization and spin.
- Virtual particles represent a weighted average over all possibilities consistent with the real-particle specifications and vertex rules. Virtual particles also "carry" a momentum which appears in mathematical expressions representing the connected line in question.
- The vertex rules (i.e., interaction rules and momentum conservation) determine whether or not a certain set of particles can collide and produce another set of particles. If a schematic can't be created for that scenario, then the scenario isn't possible. At a vertex, momentum is conserved.

• The lines of a vertex represent fields in a Lagrangian. For example, consider the Lagrangian for electrons (or positrons) interacting with an electromagnetic field.

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{EM} \tag{1}$$

where $\mathcal{L}_0$ is the part of the Lagrangian which is quadratic (this part is called the free Lagrangian) in the electron field ψ_i^e or the electromagnetic field A_μ, and $\mathcal{L}_{EM}$ is the electromagnetic interaction term

$$\mathcal{L}_{EM} = -e \sum_{i,j,k,\mu} \psi_i^{e\dagger} \gamma_{ij}^0 \gamma_{jk}^\mu A_\mu \psi_k^e \tag{2}$$

This interaction term can be seen to be a sum of products of 3 field terms, $\psi_i^{e\dagger}$, A_μ, and ψ_k^e multiplied by complex coefficients formed from products of components of γ matrices. The coefficients include an overall factor of e, which is known as the coupling constant.

- The schematic rules are obtained from the Lagrangian by one of several techniques. My favorite technique is to use the Lagrangian in a path integral whose integrand includes a term

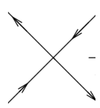
$$e^{i\int d^4x(\mathcal{L}_0+\mathcal{L}_{EM}+i\epsilon)} = e^{i\int d^4x(\mathcal{L}_0+i\epsilon)} e^{i\int d^4x \mathcal{L}_{EM}}$$
$$= e^{i\int d^4x(\mathcal{L}_0+i\epsilon)} \left(1 + i\int d^4y_1 \mathcal{L}_{EM} + i^2 \frac{1}{2!}\int d^4y_1 \mathcal{L}_{EM} \int d^4y_2 \mathcal{L}_{EM} + ...\right) \tag{3}$$

where we've Taylor expanded the second exponential, using the assumption that $\mathcal{L}_{EM}$ is small.

- What if there were an additional interaction term so that the total interaction term looks like

$$\mathcal{L}_I = \mathcal{L}_{EM} + g \sum_{i,j,k,l} \psi_i^{e\dagger} \gamma_{ij}^0 \psi_j^e \psi_k^{e\dagger} \gamma_{kl}^0 \psi_l^e. \tag{4}$$

This term is a sum of products of 4 field terms and would be represented by a vertex that looks like this.



However, such an interaction term is absent in the Lagrangian for electrons and photons. There are various kinds of rules that prohibit such a term. In general, the construction of "allowed" Lagrangians is called *model-building*.

1.2 Chapters 2 and 3

- Relativity in a nutshell
- Non-relativistic quantum mechanics in a nutshell
- The kinematics of decay and scattering: Although the Feynman diagrams are used to compute amplitudes for particles with specific momenta, actual experiments look at ranges of angles and velocities. The probability amplitudes have to be integrated appropriately over those ranges and this section covers some of those details.
- Fermi's Golden Rule: This connects ordinary quantum mechanics to the Feynman diagram approach above. We see in this section, that the collision of particles can be regarded as an interaction governed by the Hamiltonian which, in non-relativistic physics, is characterized by both a kinetic term and a potential term. The leading approximation for the collision amplitude, is obtained by an expression of the form $\langle f|\hat{H}_I|i\rangle$ where $\hat{H}_I$ is the interaction part of the Hamiltonian.

1.3 Chapter 4

This chapter begins the long slog towards adding details to the Feynman-diagram rules. Our first task is to identify the 'real particles' participating in the collision. The particle of most interest in the early days, was the electron. The standard approach to this subject, is to regard electron physics as a relativistic extension to the free-particle Schrodinger equation.

- The Schrodinger equation violates Lorentz invariance. The simplest extension is the Klein-Gordon equation. Unfortunately, it doesn't describe any particles known in the 1920's. However, the K-G equation was apparently discarded on the basis of some unresolved conceptual issues, rather than on experimental grounds.
- An alternative, discovered by Dirac, was the Dirac equation. This required the hypothesis that electrons had to be represented by multi-component functions of time and space (unlike Schrodinger particles that were one-component complex functions of time and space). That was a happy observation, because experiments had already concluded that electrons appear to have an extra degree of freedom which was called "spin".

- Dirac ended up having to sort out the same conceptual difficulties that had plagued the Klein-Gordon equation, but because of the fortuitous explanation for spin, he fought his way through all that. This ultimately led him to a more systematic interpretation of his equation as an equation for fields, rather than for states.

- In Chapter 4, we are introduced to the notations used to characterize Dirac particles in terms of their momentum and spin (Dirac spinors) as well as to describe kinematic and frame-transformation properties (Dirac gamma matrices). This is all essential for properly identifying the multiplicands that appear for the fermion lines in Feynman diagrams.

1.4 Where we're going

- We're now in a position to provide a complete set of rules for Feynman diagrams describing electron, positron and photon scattering. We haven't talked about the photon so we start there. But then – rather than dwell on the derivation of the rules – we can simply take for granted that the path-integral formulation, along with rules for moments of integrals involving exponents of quadratic forms, will lead to those rules. This is Chapter 5.

- Rules are messy. Some examples are called for. This is Chapter 6.

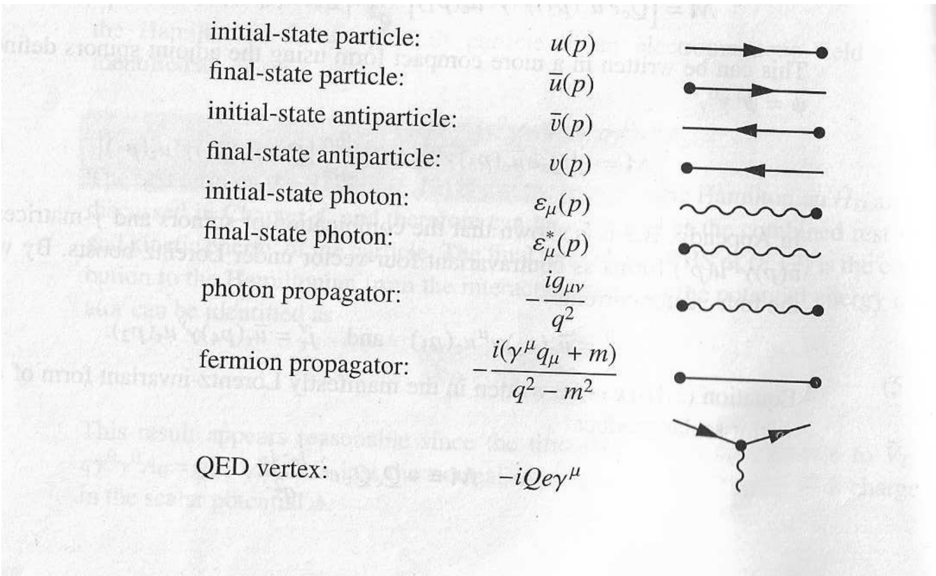
- After that, we are ready to start introducing other particles and their Feynman rules. We start with the proton. On the one hand, one can treat it just like a heavier cousin of the electron. On the other hand, we've learned that the proton – unlike the electron – is a composite particle whose components are quarks. So an electron-proton collision is actually a collision of an electron with 3 bound quarks. Chapter 7 discusses the kinds of modifications to be made when a particle (such as the proton) has structure. In practice, this is the sort of thing that helped physicists establish experimentally the quark structure of the proton.

- Chapter 8 goes further into this topic but, in my opinion, is somewhat more detailed that we need to cover. It might be of interest to those who want to understand more about the experiments that probe the deep structure of protons.

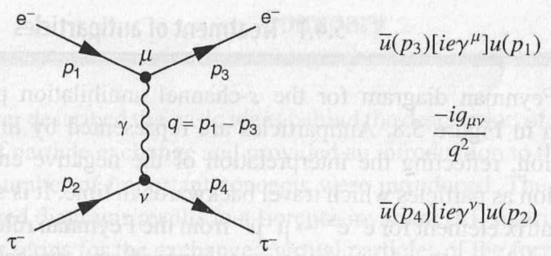
- Finally in Chapter 9, we can begin the analysis of quark symmetries and more generally, the symmetry-theory underlying the taxonomy of particles discussed in Chapter 1. Thomson's approach is grounded in the experimental evidence that led to our current models. Another approach would be more formal and elegant where we begin by postulating certain symmetries and then using group properties to derive patterns and interactions.

2 Chapter 5: Feynman rules for QED (Quantum Electrodynamics)

We construct Feynman diagrams by joining permissible vertices to one another, and to "free lines" (real particles) representing the initial or final particles of a scattering or decay process. Each free line is associated with a particular spin or polarization and all lines are labelled by a momentum. Momentum is conserved at vertices. The rules for the Feynman diagram tell you how to compute the amplitude.



Consider the example of an electron scattering with a tau (a third-generation charged lepton).



The Feynman diagram for the QED scattering process $e^-\tau^- \to e^-\tau^-$ and the associated elements of the matrix element constructed from the Feynman rules. The matrix element is comprised of a term for the electron current, a term for the tau-lepton current and a term for the photon propagator.

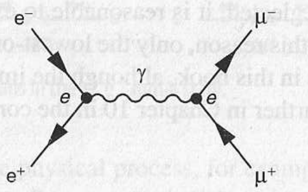
We put all this together to compute the Lorentz invariant matrix element proportional to the scattering amplitude.

$$-i\mathcal{M} = [\bar{u}(p_3)\{ie\gamma^\mu\}u(p_1)]\frac{-ig_{\mu\nu}}{q^2}[\bar{u}(p_4)\{ie\gamma^\nu\}u(p_2)]. \tag{5}$$

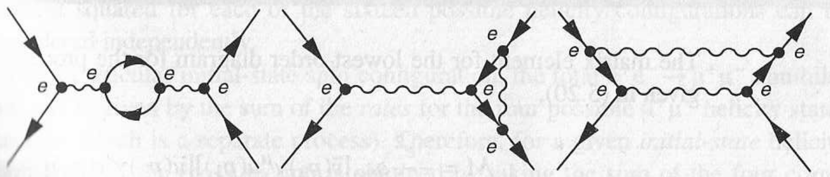
The above diagram is "interaction by particle exchange" and is sometimes called a t-channel diagram because the virtual particle has a momentum $q = p_1 - p_3$, and q^2 is known as the Mandelstam variable "t".

3 Chapter 6: Electron-positron annihilation

The process of electron-positron annihilation, in leading perturbative order, has a similar diagram to the particle-exchange process above. There are two vertices joined by a connected line (virtual particle). However, the time-orientation is different. Not surprisingly, the amplitudes are related.



The lowest-order Feynman diagram for the QED annihilation process $e^+e^- \rightarrow \mu^+\mu^-$.



Three of the $O(\alpha^4)$ Feynman diagrams contributing the QED annihilation process $e^+e^- \rightarrow \mu^+\mu^-$.

The diagram on top has two vertices and is a sideways version of the diagram for electron-tau scattering. The diagrams below illustrate higher orders in perturbation theory. They each have 4 vertices (and are therefore of order 4 in the coupling constant).

Following the Feynman rules, we obtain

$$\mathcal{M} = -\frac{e^2}{q^2} g_{\mu\nu} [\bar{v}(p_2)\gamma^\mu u(p_1)][\bar{u}(p_3)\gamma^\nu v(p_4)], \tag{6}$$

where the incoming particles have momenta p_1 and p_2, and the momentum q of the virtual photon, is $q = p_1 + p_2$, the center-of-mass 4-momentum. Using Mandelstam variables, we see that $s = q^2$, so we often describe this process as an s-channel process.

The text goes into a lot of detail about how to related this amplitude to quantities that are measured in actual experiments. For example, rather than measuring the spin – which is used for obtaining the above amplitude – experimentalists often describe scattering in terms of helicities. The text shows the relationship between spin and helicity. Furthermore, experiments are sometimes unable to distinguish between one spin (or helicity) and another, in which case we are interested in the total amplitude obtained by averaging or summing over all spins. Again, the text describes some convenient relationships for obtaining those without having to struggle through messy 4 x 4 matrix calculations.