Another way of computing decay ratios

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June 21, 2022

In my June notes on Thomson Chapter 9, I showed how to compute decay ratios by constructing an isospin-invariant Lagrangian describing interactions between Δ-particles, nuleons and pions.

There is another way of proceeding, and probably what Thomson had in mind when posing problem 9.3. The Born approximation can be used to compute the transition from one state to another. In particular, suppose the initial state represents a Δ^{++} particle, and the final state represents the particle-pair consisting of a proton and π^+. Then the Born approximation for the decay amplitude, is proportional to

$$\langle \Delta^{++} | H_I | p\pi^+ \rangle \tag{1}$$

where H_I is the isospin-invariant interaction Hamiltonian.

Now, isospin invariance is described mathematically by the equation

$$U(\alpha, \hat{u})^\dagger H_I U(\alpha, \hat{u}) = H_I \tag{2}$$

where $U(\alpha, \hat{u})$ is an isospin transformation parameterized by an angle α and an axis $\hat{u}$.

This invariance-equation can be plugged into the Born approximation to obtain

$$\langle \Delta^{++} | U(\alpha, \hat{u})^\dagger H_I U(\alpha, \hat{u}) | p\pi^+ \rangle = \langle \Delta^{++} | H_I | p\pi^+ \rangle. \tag{3}$$

This equation can be re-organized so that it reads

$$\langle \psi_i | H_I | \psi_f \rangle = \langle \Delta^{++} | H_I | p\pi^+ \rangle \tag{4}$$

where $|\psi_i >= U(\alpha, \hat{u}) |\Delta^{++}\rangle$ and $|\psi_f >= U(\alpha, \hat{u}) |p\pi^+\rangle$.

In other words, eq. (4) says, in plain English, "the decay amplitude of $|\Delta^{++}\rangle$ to a proton and a positive pion, is the same as the decay amplitude of the states $|\psi_i\rangle$ to the state $\psi_f\rangle$." Our goal is, for example, to pick the right parameters α and $\hat{u}$ so that the isospin transformation changes $|\Delta^{++}\rangle$

to $\beta_1|\Delta^0\rangle$, and also changes $|p\pi^+\rangle$ to $\beta_2|p\pi^+\rangle$. There's no a priori guarantee that such a transformation can be found. However, it turns out that it can, and that the proportionality constants β_1 and β_2 have the property $|\beta_1\beta_2| = \sqrt{3}$. In other words,

$$|\langle\Delta^{++}|H_I|p\pi^+\rangle| = |\langle\psi_i|H_I|\psi_f\rangle| = \sqrt{3}|\langle|\Delta^0|H_I|p\pi^-\rangle|. \tag{5}$$

When we square the magnitudes, we see that the decay rate on the left is 3 times the decay rate on the right.

Even if this seems straightforward, the remaining details involve the representations of the isospin transformations. Frankly, I can't think of an easy systematic way to figure out what isospin parameters can be used to make the desired transformation. I've stared at some isospin-1 matrices but nothing popped out at me.