# Are quark bound states *'real'* particles?

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## 1 Reality

We've analyzed the possible energy (mass) eigenstates of an SU(3)-invariant theory. These correspond to irreducible representations of SU(3). A particularly fruitful model of particle masses is one where we regard the hadrons as bound states of quarks. The resultant energy/mass eigenstates are then automatically grouped into irreducible SU(3) representations with common (aka 'degenerate') eigenvalues. For example, the baryons are regarded as bound states of three quarks, and the detailed baryon properties have to do with the flavors, spin-orientations and orbital configurations of the quarks in their bound states.

It's been noted that the above model is only an approximation. First of all, the quarks have different masses (thus violating strict flavor symmetry) and even though their interactions are flavor-independent, the bound-state energies depend on the quark masses and thus aren't precisely the same for all quark combinations.

Secondly, the idea of a bound state comes from intuitions about the hydrogen atom, whose properties are derived using non-relativistic quantum mechanics. In particular, when solving equations for the hydrogen atom, we assume that the proton and electron are the only particles binding together. In a relativistic theory, particles can be created and annihilated. So the situation is somewhat similar to what would happen if you were trying to compute earth's orbit around the sun while taking into account earth's collisions with huge asteroids. In that case, orbits are only approximately elliptical and over a long period of time, earth would fall into the sun or alternatively be kicked out of the solar system. Similarly, when looking at quark-binding, one can end up with temporary bound states, aka 'unstable' bound states.

So, are the quark bound states *'real'*? This is a deeply philosophical question that requires a definition of *'real'*. I don't want to attempt a proper analysis of this question. It's been a subject of interest to the greatest philosophers through the ages. However, I'll toss out two somewhat opposing views on the matter.

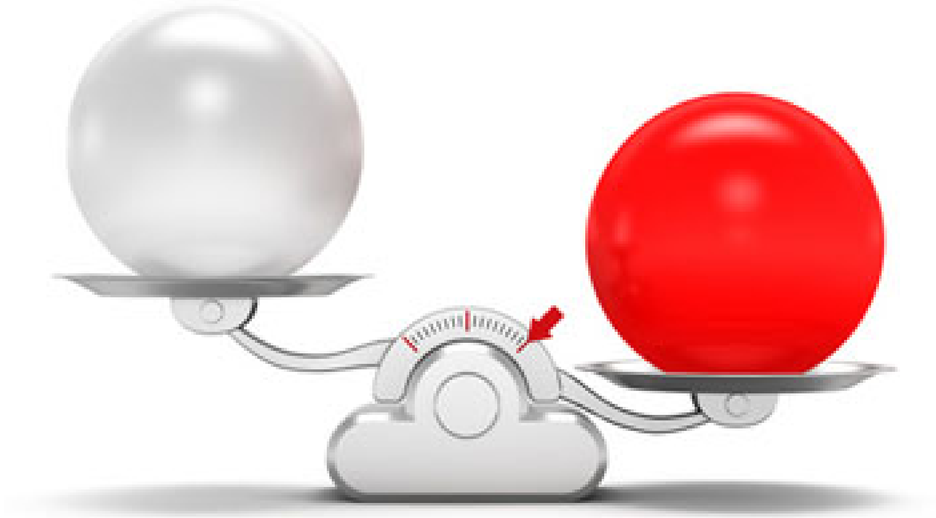
- Heisenberg's view: The only things which are 'real' are those capable of being measured ('observed'). Famously, Heisenberg argued that there is no reality to the question "what is the simultaneous momentum and position of a particle?", since no measurement can be done which can determine position without disturbing momentum, etc.

- Einstein's view (sort of): There is a relatively-simple mathematical model so that every measurement is uniquely determined by the parameters of the model. This mathematical model is called 'reality'. So if you first measure the position of a particle and subsequently measure its momentum, the results are uniquely predicted by a single mathematical model of the world with particular values for the parameters of that model.

Heisenberg's view is blemished by the imprecise notion of what constitutes a measurement. For example, is your consciousness part of the measurement? Einstein's view is blemished by the imprecise notion of 'relatively-simple'.

Both views are resolved in practice by acknowledging the role of approximation in science and engineering. What Heisenberg's principle demonstrated, was that different kinds of approximations were required for microscopic/quantum phenomena, than were appropriate for macro/classical phenomena.

So, what do we mean by a 'real particle'? When the particle is large (macroscopic) it has measurable properties that we can all agree on.



When the particle is microscopic, its properties are inferred from its collisions with other particles. As an example, free electrons can be produced through chemical reactions on separated plates, and the electron trajectories are inferred through collisions with gas etc. More specifically to particle physics, electrons or protons can be accelerated and then collided (scattered) with one another to produce a variety of other particles that can be deflected in a particular direction dependent on the masses of those particles. Those new particles can in turn be scattered, etc. However, from a mathematical point of view, the new particles don't 'precisely' have the property of mass. Rather, the electromagnetic deflection can be computed by assuming an 'approximate' mass for those particles.

Consider, for example, a collision of the form

$$e^+ \oplus e^- \to \aleph \to \text{stuff} \tag{1}$$

where an electron and a positron produce an $\aleph$ 'particle' which then decomposes into a particular collection of other particles which we call 'stuff'. In this process, we don't directly see the $\aleph$ 'particle' because it decays into 'stuff', but we can infer its existence by measuring the rate of production of 'stuff' as a function of the center-of-mass energy of the electron-positron pair. An approximate prediction for this rate, is obtained from the collision amplitude, proportional to the Feynman propagator as we've discussed previously.

$$G_F(p) = \frac{i}{p^2 - m^2} \tag{2}$$

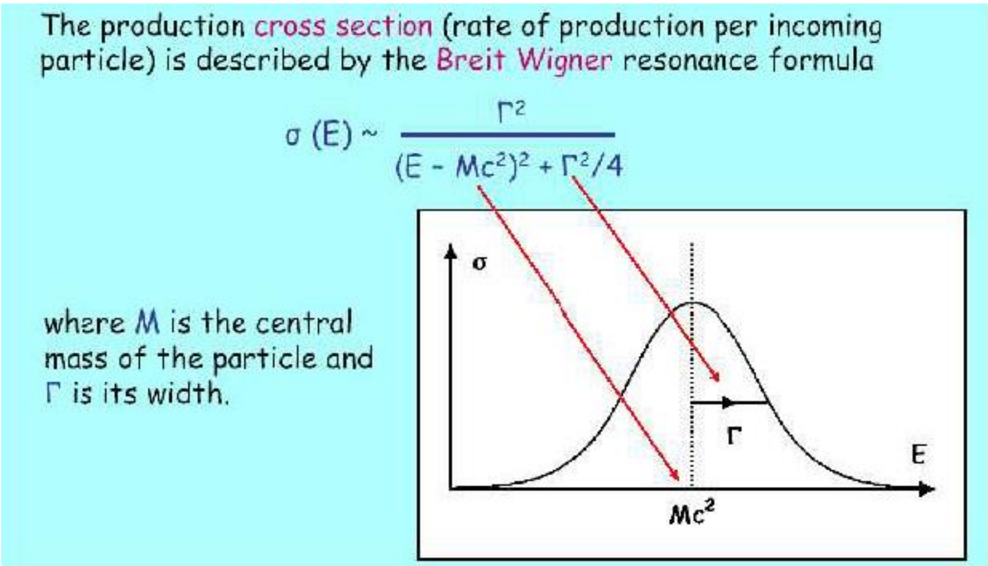
where $p$ is the 4-momentum of the electron-positron pair. In the center-of-mass frame, $p = (2E, \mathbf{0})$. A more precise approximation is obtained by considering perturbative corrections to the Feynman propagator, which can be shown to have the form

$$\Delta_F(p) = \frac{i}{p^2 - m^2 + im\Gamma_\aleph} \tag{3}$$

It turns out that $\Gamma_\aleph$ is proportional to the rate of decay of the $\aleph$ particle.The total scattering rate is then proportional to the magnitude-squared of $\Delta_F(p)$ and we end up with the cross-section (rate)

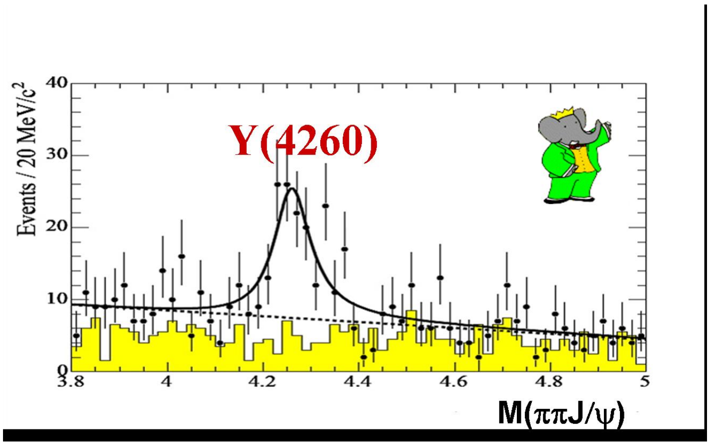
$$\sigma(e^+ \oplus e^- \to \aleph \to \text{stuff}) \propto \frac{1}{(4E^2 - m^2)^2 + m^2\Gamma_\aleph^2}. \tag{4}$$

This is known as the Breit-Wigner resonance formula.



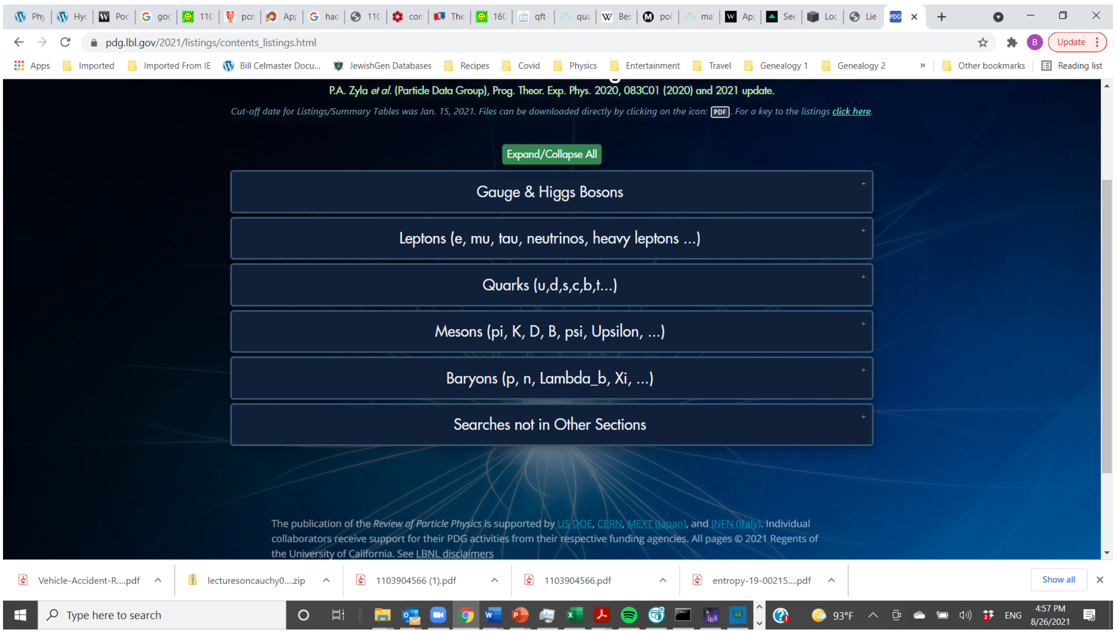

Note that the width of the curve, $\Gamma_\aleph$, is the imaginary part of the denominator in the propagator.

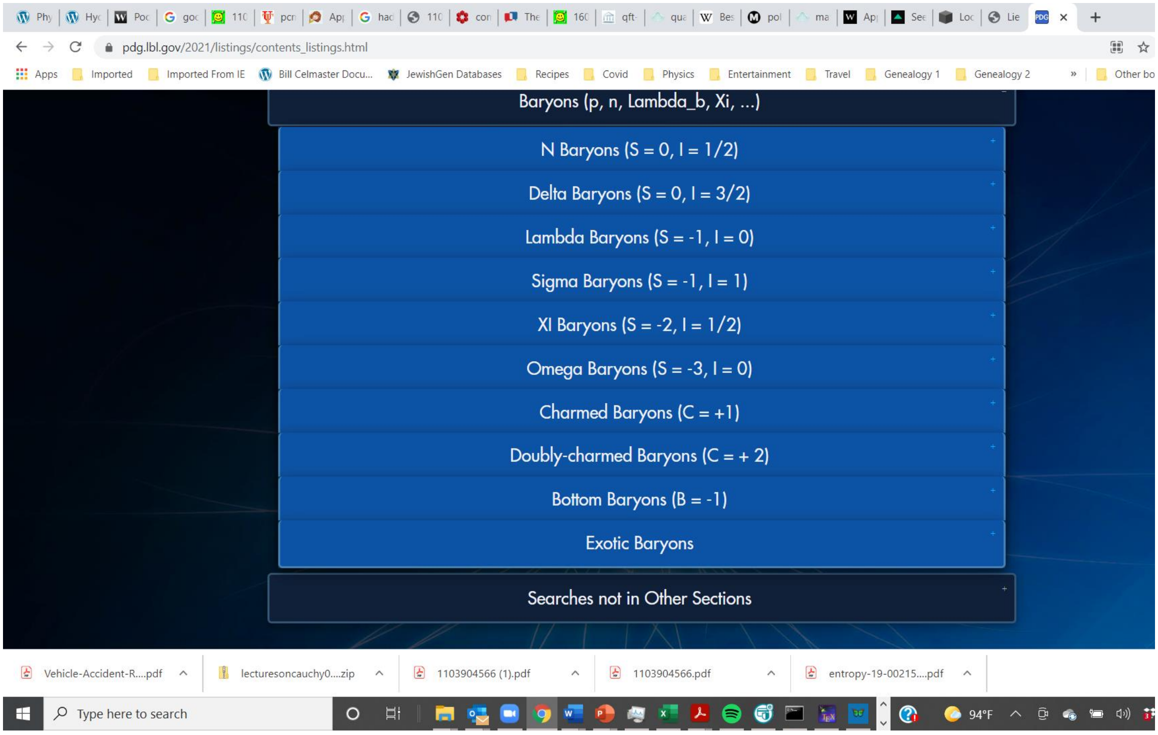
A real-life example of this kind of thing is shown here, in the production of a mysterious particle known as Y(4260), speculated to be a 4-quark bound state.

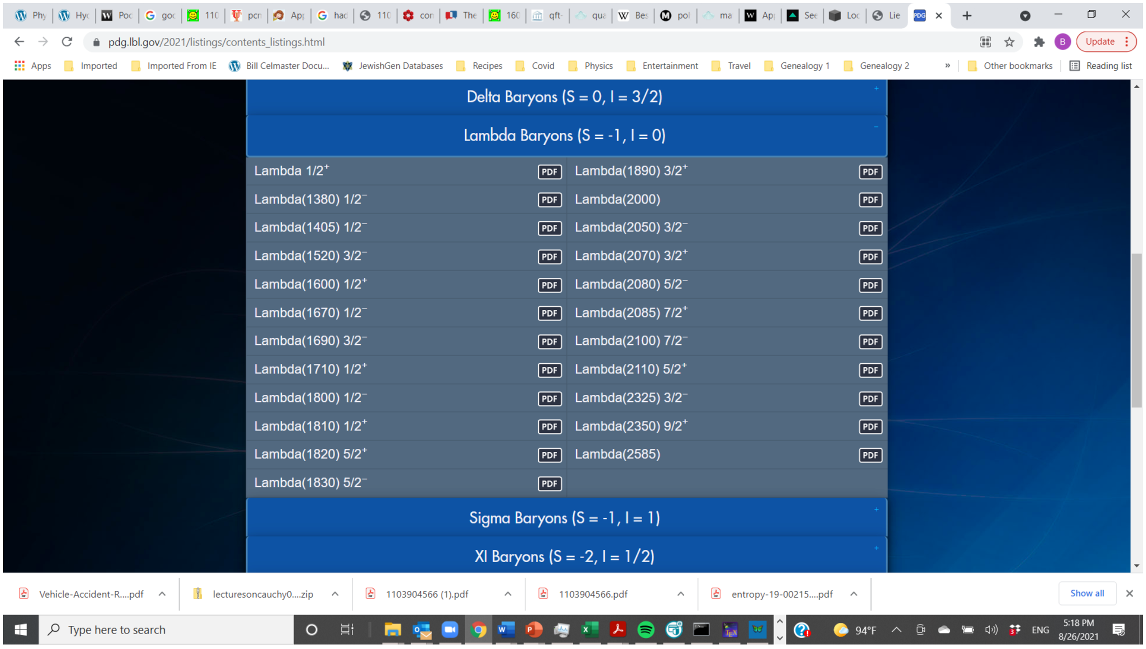



Can we imagine taking two unstable particles and colliding them? Yes we can, but the process is a bit convoluted. First you have to create the particles – often requiring the collision of other particles (e.g. electrons). Then the new particles would need to be separated from other by-products of the original collision. This might be accomplished by collimation of some sort. This multi-step process could be modelled using various approximations that apply to reasonably-stable 'particles'. In the end, the approximation ends up looking a lot like the computation for collisions of two stable particles, but with perturbations.

# 2 The particle table from The Particle Data Group




pdg.lbl.gov/2021/listings/contents_listings.html
Baryons (p, n, Lambda_b, Xi, ...)
N Baryons (S = 0, I = 1/2)
Delta Baryons (S = 0, I = 3/2)
Lambda Baryons (S = -1, I = 0)
Sigma Baryons (S = -1, I = 1)
XI Baryons (S = -2, I = 1/2)
Omega Baryons (S = -3, I = 0)
Charmed Baryons (C = +1)
Doubly-charmed Baryons (C = + 2)
Bottom Baryons (B = -1)
Exotic Baryons
Searches not in Other Sections



pdg.lbl.gov/2021/listings/contents_listings.html

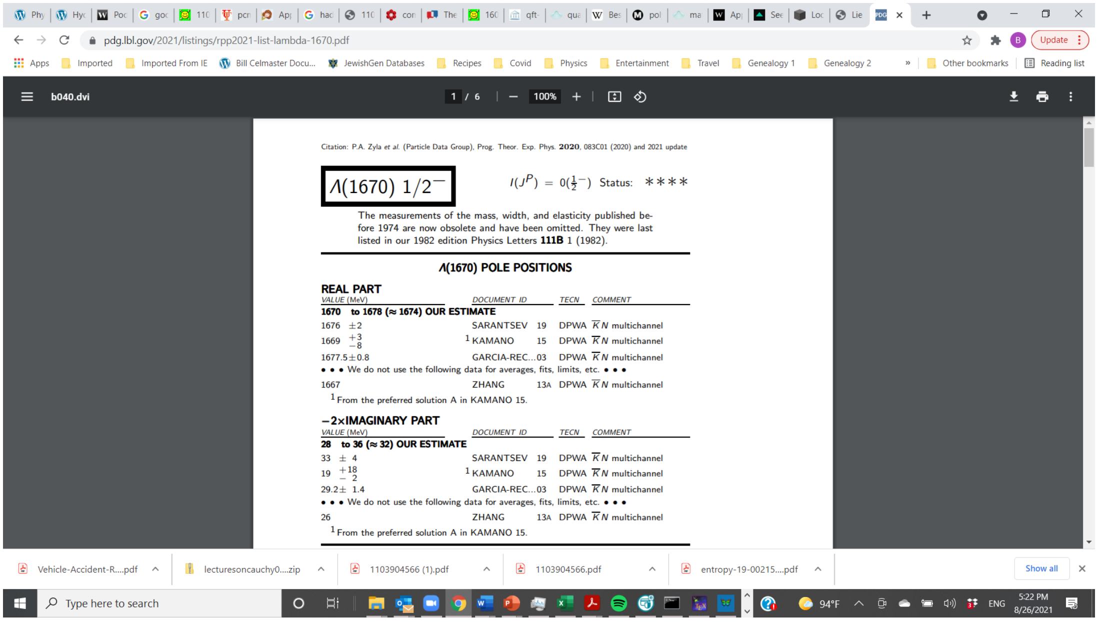
Delta Baryons (S = 0, I = 3/2)

Lambda Baryons (S = -1, I = 0)

| | |
|---|---|
| Lambda $1/2^+$ PDF | Lambda(1890) $3/2^+$ PDF |
| Lambda(1380) $1/2^-$ PDF | Lambda(2000) PDF |
| Lambda(1405) $1/2^-$ PDF | Lambda(2050) $3/2^-$ PDF |
| Lambda(1520) $3/2^-$ PDF | Lambda(2070) $3/2^+$ PDF |
| Lambda(1600) $1/2^+$ PDF | Lambda(2080) $5/2^-$ PDF |
| Lambda(1670) $1/2^-$ PDF | Lambda(2085) $7/2^+$ PDF |
| Lambda(1690) $3/2^-$ PDF | Lambda(2100) $7/2^-$ PDF |
| Lambda(1710) $1/2^+$ PDF | Lambda(2110) $5/2^+$ PDF |
| Lambda(1800) $1/2^-$ PDF | Lambda(2325) $3/2^-$ PDF |
| Lambda(1810) $1/2^+$ PDF | Lambda(2350) $9/2^+$ PDF |
| Lambda(1820) $5/2^+$ PDF | Lambda(2585) PDF |
| Lambda(1830) $5/2^-$ PDF | |

Sigma Baryons (S = -1, I = 1)

XI Baryons (S = -2, I = 1/2)



Citation: P.A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020) and 2021 update

$\boxed{\Lambda(1670)\ 1/2^-}$ $I(J^P) = 0(\tfrac{1}{2}^-)$ Status: ∗∗∗∗

The measurements of the mass, width, and elasticity published before 1974 are now obsolete and have been omitted. They were last listed in our 1982 edition Physics Letters **111B** 1 (1982).

## $\Lambda$(1670) POLE POSITIONS

### REAL PART

| *VALUE* (MeV) | *DOCUMENT ID* | | *TECN* | *COMMENT* |
|---|---|---|---|---|
| **1670 to 1678 (≈ 1674) OUR ESTIMATE** | | | | |
| 1676 $\pm 2$ | SARANTSEV | 19 | DPWA | $\overline{K}N$ multichannel |
| 1669 $^{+3}_{-8}$ | [1] KAMANO | 15 | DPWA | $\overline{K}N$ multichannel |
| 1677.5 $\pm$ 0.8 | GARCIA-REC... | 03 | DPWA | $\overline{K}N$ multichannel |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | |
| 1667 | ZHANG | 13A | DPWA | $\overline{K}N$ multichannel |

[1] From the preferred solution A in KAMANO 15.

### −2×IMAGINARY PART

| *VALUE* (MeV) | *DOCUMENT ID* | | *TECN* | *COMMENT* |
|---|---|---|---|---|
| **28 to 36 (≈ 32) OUR ESTIMATE** | | | | |
| 33 $\pm$ 4 | SARANTSEV | 19 | DPWA | $\overline{K}N$ multichannel |
| 19 $^{+18}_{-2}$ | [1] KAMANO | 15 | DPWA | $\overline{K}N$ multichannel |
| 29.2 $\pm$ 1.4 | GARCIA-REC... | 03 | DPWA | $\overline{K}N$ multichannel |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | |
| 26 | ZHANG | 13A | DPWA | $\overline{K}N$ multichannel |

[1] From the preferred solution A in KAMANO 15.