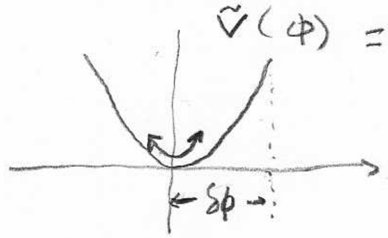
Classical $\quad E = \frac{1}{2}mv^2 + V(x)$

Field Theory $\quad E = \sum_\mu \frac{1}{2}\frac{\partial\phi}{\partial x^\mu}\frac{\partial\phi}{\partial x_\mu} + \tilde{V}(\phi)(x)$

① Definition of mass in field theory:



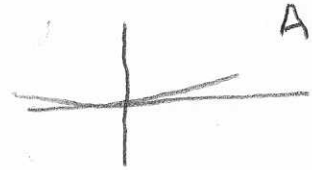
$$\tilde{V}(\phi) = V_0 + \frac{1}{2}\boxed{\frac{\partial^2 V}{\partial\phi^2}}(\delta\phi)^2 + \frac{1}{4!}\frac{\partial^4 V}{\partial\phi^2}(\delta\phi)^4 + \dots$$

↳ $\boxed{\text{mass}}$

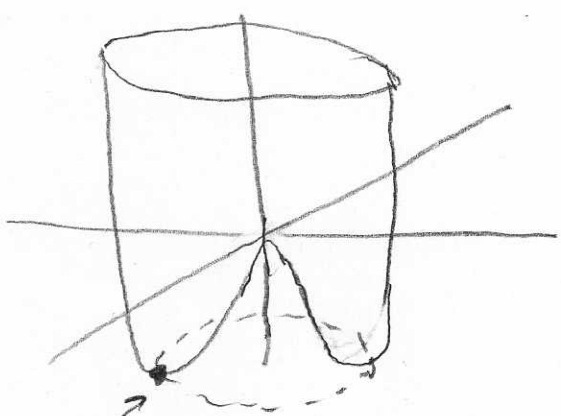
mass is the quadratic term in the Taylor expansion around the min.

bigger mass

Almost 0 mass



② What if there are multiple minima? Pick one. Expand around it:



$(\phi_1, \phi_2) = (v, 0)$

e.g. $\tilde{V}(\phi) = -2v^2\phi^2 + \lambda\phi^4$

This direction has a wall. ⟺ MASS ≠ 0

Along the dotted line the path is flat ⟺ MASS = 0

How do we see this in equations?

Write $\phi = (\phi_1' - v, \phi_2')$. Then

$$\tilde{V}(\phi) = -2v^2 \left[(\phi_1' - v)^2 + \phi_2'^2 \right] + \left[(\phi_1' - v)^2 + \phi_2'^2 \right]^2$$

$\boxed{\text{when } \phi = (v, 0),\ \phi_1' = 0 = \phi_2' = 0}$ $\boxed{\text{That's a MINIMUM}}$

Expand $\tilde{V}$ around 0 → POSITIVE MASS

$$\tilde{V} = \text{constant} \boxed{+ 4v^2 \phi_1'^2} + O(\phi_1'^3) + O(\phi_2'^3)$$

$\boxed{\text{NO Quadratic } \phi_2' \text{ term}}$ $\Rightarrow$ ϕ_2' has 0-mass

Suppose we have a gauge field, B_μ

$$\tilde{V} \sim \ldots + g^2 B_\mu B^\mu (\phi_1^2 + \phi_2^2)$$

$\boxed{\text{NO QUADRATIC TERMS FOR } B \Rightarrow \text{0-mass}}$

But write $(\phi_1, \phi_2) = (\phi_1' - v, \phi_2')$,

then $\tilde{V} \sim \ldots + \boxed{g^2 v^2 B_\mu B^\mu} + \ldots$

MASS TERM