Experimental consequences of the electroweak Lagrangian

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1 Introduction

The PHYSICS method (aka 'scientific method'):

- Find 'the fundamental laws'. e.g. quantum mechanics, Lorentz invariance, SU(3) invariance, Maxwell's equations etc., the Action principle (leading to Lagrange's equations in classical mechanics), etc.
- Calculational methods for deriving consequences of the laws. e.g. Feynman diagrams
- Comparison with experiments

The steps of this method are tightly intertwined. Generally experiments lead to laws, and then the laws are tested on other experiments. But without calculational methods, neither direction is possible so there are often extended periods of time after new laws are hypothesized, where calculational methods are developed from them.

Over the past 120 years or so, starting with the law of Lorentz invariance, one of the favorite calculational method for inferring new laws, has been the use of groups (symmetries) and their linear representations:

$$D(g_1)D(g_2)v = D(g_1 \circ g_2)v \tag{1}$$

where $g_1 \circ g_2$ denotes the composition of two group elements (e.g. a rotation followed by a boost), v is a vector in a vector space (the representation space) and $D(g_1)$ is a matrix associated with the group element g_1.

The standard model is the current best theory of the fundamental symmetry groups of nature, their representations as particles and their laws of interaction governed by field theory.

Field theory can either be regarded as a combination of the action principle combined with canonical commutation relations that create a bridge from particles to fields **or** as a path integral over fields whose Green functions create a bridge from particles to moments of the path integral. In both cases, the key mathematical encapsulation of the theory, is the **Lagrangian**.

Ingredients of the standard model (as implemented by the Lagrangian):

- Lorentz invariance
- Local SU(3) invariance [strong interactions]
- Local SU(2) x U(1) invariance [weak interactions and electromagnetism]
- Broken SU(2) x U(1) symmetry (the Lagrangian is symmetric but the observer has to pick a preferred vacuum)
- Bosons
  - (Lorentz-) Scalar boson – the Higgs particle
  - (Lorentz-)Vector bosons
    - Gluons – adjoint representation of SU(3). **The symmetry isn't broken, so all 8 gluons are simply designated as A^a.**
    - Electroweak bosons – adjoint representation of SU(2) x U(1). **The symmetry is broken, so all 4 electroweak bosons are given different names: W^+, W^-, Z, A, the last of these being the photon.**
- Spin-$\frac{1}{2}$ Fermions – quarks, charged leptons (electron, muon, tau), neutral leptons (electron-neutrino, muon-neutrino, tau-neutrino). Each of these transform in irreducible representations of SU(3) and SU(2)

The history of weak-interaction physics mostly has to do with this last item (the fermions), much of which is covered in Thomson chapters 11-14. However, once the electroweak model had been developed by Weinberg and others, people began studying interactions of $W^{\pm}$ and Z mesons, as well as the Higgs bosons, whose properties and scattering amplitudes are discussed in Thomson chapters 15-17.

We will introduce the Lagrangians for weak interactions. As a reminder, we had derived the strong-interaction Lagrangian by demanding local SU(3) invariance, and obtained

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}(F^a_{\mu\nu})^2 - \frac{1}{2\xi}(\partial_\mu A^a_\mu)^2 + (\partial_\mu \bar{c}^a)(\delta^{ac}\partial_\mu + g f^{abc} A^b_\mu)c^c \\ &+ \bar{\psi}_i(\delta_{ij} i\not{\partial} + g\not{A}^a T^a_{ij} - m\delta_{ij})\psi_j \\ &+ \left[(\delta_{ki}\partial_\mu - igA^a_\mu T^a_{ki})\phi_i\right]^\star \left[(\delta_{kj}\partial_\mu - igA^a_\mu T^a_{kj})\phi_j\right] - M^2\phi^\star_i\phi_i, \end{aligned} \tag{26.1}$$

where c^a and $\bar{c}^a$ are the Faddeev–Popov ghosts and anti-ghosts respectively and

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu. \tag{26.2}$$

The group-properties are contained in the structure constants f^{abc} and the matrices T^a_{ij}. The form of this strong-interaction Lagrangian looks fairly simple partly because the gluon fields are all indexed – e.g. A^μ_a. By contrast, the SU(2) x U(1) Lagrangian terms look considerably more complicated, largely because each vector boson is separately identified. In addition, the SU(2) x U(1) Lagrangian has a much richer fermion sector which includes not only quarks but also leptons. We'll start with a discussion of the SU(2) x U(1) representations of fermions.

2 Fermion kindergarten

Here is a review of some basic fermion facts.

2.1 2-component spinor massless Dirac equations; i.e. Weyl equations

Define two 2-component quantities (usually called spinors but think of them as 2-vectors in a vector space). For simplicity, imagine we live in a 2D spacetime $x = (t, z)$

$$\psi_R(x) = \begin{pmatrix} \psi_R^1(x) \\ \psi_R^2(x) \end{pmatrix} \quad \psi_L(x) = \begin{pmatrix} \psi_L^1(x) \\ \psi_L^2(x) \end{pmatrix} \tag{2}$$

We declare that they satisfy two separate equations which we call the massless Dirac equations or more commonly, the *Weyl equations.*

$$\begin{pmatrix} (i\partial_t + i\partial_z)\psi_R^1(x) \\ (i\partial_t - i\partial_z)\psi_R^2(x) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \tag{3}$$

$$\begin{pmatrix} (i\partial_t - i\partial_z)\psi_L^1(x) \\ (i\partial_t + i\partial_z)\psi_L^2(x) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \tag{4}$$

If we can find a linear transformation on ψ_R etc., which preserves the form of the Dirac equation under a change of reference frame, then we say the Dirac equation is Lorentz invariant. We will show how this works for the *boost* transformations. An example of a boost transformation is the change of coordinates and fields between a reference frame at rest, and another reference frame moving at constant velocity v. The key requirement for these transformation laws, is that they follow the group composition rules for the Lorentz group.

Propose the following transformation law for boosts of ψ_R and ψ_L.

$$\psi_R(\mathbf{x}') = \begin{pmatrix} e^{-\frac{\beta}{2}}\psi_R'^1 \\ e^{\frac{\beta}{2}}\psi_R'^2 \end{pmatrix}(\mathbf{x}') \quad \psi_L(\mathbf{x}') = \begin{pmatrix} e^{\frac{\beta}{2}}\psi_L'^1 \\ e^{-\frac{\beta}{2}}\psi_L'^2 \end{pmatrix}(\mathbf{x}'). \tag{5}$$

where $\mathbf{x}' = (t', z') = (t\cosh\beta + z\sinh\beta, t\sinh\beta + z\cosh\beta)$ and $\beta = \tanh^{-1}(v)$.

Start by applying the chain rule to the right-handed Weyl eq. (3). For example,

$$
\begin{aligned}
\partial_t \psi_R(x) = \frac{\partial}{\partial t}\psi_R(x) &= \left(\frac{\partial t'}{\partial t}\frac{\partial}{\partial t'} + \frac{\partial z'}{\partial t}\frac{\partial}{\partial z'}\right)\psi_R(x') \\
&= \left(\cosh\beta\frac{\partial}{\partial t'} + \sinh\beta\frac{\partial}{\partial z'}\right)\psi_R(x')
\end{aligned} \tag{6}
$$

Then apply the field transformation equation (5) to get

$$
\begin{pmatrix} e^{\frac{\beta}{2}} & 0 \\ 0 & e^{-\frac{\beta}{2}} \end{pmatrix} \begin{pmatrix} (i\partial_{t'} + i\partial_{z'})\,\psi_R'^1(\mathbf{x'}) \\ (i\partial_{t'} - i\partial_{z'})\psi_R'^1(\mathbf{x'}) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \tag{7}
$$

Dividing by the first matrix, we end up with the right-handed Weyl equation (3) again, so we've now shown that the equation is boost-invariant.

Similarly, we can show that the left-handed Weyl equation (4) is boost-invariant.

2.2 Dirac mass terms

So far, these equations don't have mass terms – i.e., terms without derivatives. Suppose we try to extend the right-handed Weyl equation to:

$$
\begin{pmatrix} (i\partial_t + i\partial_z)\psi_R^1(x) \\ (i\partial_t - i\partial_z)\psi_R^2(x) \end{pmatrix} = m \begin{pmatrix} \psi_R^1(x) \\ \psi_R^2(x) \end{pmatrix}. \tag{8}
$$

Then apply a boost transformation to obtain

$$
\begin{pmatrix} e^{\frac{\beta}{2}} & 0 \\ 0 & e^{-\frac{\beta}{2}} \end{pmatrix} \begin{pmatrix} (i\partial_{t'} + i\partial_{z'})\,\psi_R'^1(\mathbf{x'}) \\ (i\partial_{t'} - i\partial_{z'})\psi_R'^1(\mathbf{x'}) \end{pmatrix} = \begin{pmatrix} e^{-\frac{\beta}{2}} & 0 \\ 0 & e^{\frac{\beta}{2}} \end{pmatrix} m \begin{pmatrix} \psi_R'^1(x') \\ \psi_R'^2(x') \end{pmatrix}. \tag{9}
$$

This is **NOT** the same form of equation as eq. (8). The diagonal matrices differ on the left and right sides so this equation is **NOT** Lorentz invariant. However, remember that

$$
\psi_L(\mathbf{x'}) = \begin{pmatrix} e^{\frac{\beta}{2}}\psi_L'^1 \\ e^{-\frac{\beta}{2}}\psi_L'^2 \end{pmatrix}(\mathbf{x'}) = \begin{pmatrix} e^{\frac{\beta}{2}} & 0 \\ 0 & e^{-\frac{\beta}{2}} \end{pmatrix} \begin{pmatrix} \psi_L'^1(x') \\ \psi_L'^2(x') \end{pmatrix} \tag{10}
$$

So if we replace eq. (8) by

$$\begin{pmatrix}(i\partial_t + i\partial_z)\psi_R^1(x)\\(i\partial_t - i\partial_z)\psi_R^2(x)\end{pmatrix} = m\begin{pmatrix}\psi_L^1(x)\\\psi_L^2(x)\end{pmatrix}, \tag{11}$$

then the boosted equation will be

$$\begin{pmatrix}e^{\frac{\beta}{2}} & 0\\0 & e^{-\frac{\beta}{2}}\end{pmatrix}\begin{pmatrix}(i\partial_{t'} + i\partial_{z'})\,\psi_R'^1(\mathbf{x}')\\(i\partial_{t'} - i\partial_{z'})\psi_R'^1(\mathbf{x}')\end{pmatrix} = \begin{pmatrix}e^{\frac{\beta}{2}} & 0\\0 & e^{-\frac{\beta}{2}}\end{pmatrix} m\begin{pmatrix}\psi_L'^1(x')\\\psi_L'^2(x')\end{pmatrix}. \tag{12}$$

which is indeed the same form as eq. (11). We've restored Lorentz invariance, but we now require both ψ_R and ψ_L in a single equation. In other words, our massive theory must have 4, instead of 2, components.

We end up with a combination of equations that look like this:

$$\begin{aligned}\begin{pmatrix}(i\partial_t + i\partial_z)\psi_R^1(x)\\(i\partial_t - i\partial_z)\psi_R^2(x)\end{pmatrix} &= m\begin{pmatrix}\psi_L^1(x)\\\psi_L^2(x)\end{pmatrix}\\\begin{pmatrix}(i\partial_t - i\partial_z)\psi_L^1(x)\\(i\partial_t + i\partial_z)\psi_L^2(x)\end{pmatrix} &= m\begin{pmatrix}\psi_R^1(x)\\\psi_R^2(x)\end{pmatrix}.\end{aligned} \tag{13}$$

That's the massive Dirac equation, but written a bit differently than we've done before. In the usual Dirac equation, we have 4 components indexed from 1 through 4 whereas in the above equations, our 4 components are written as two 2-spinors labelled L and R and each with components 1 and 2.

IT TURNS OUT IN MODERN WEAK-INTERACTION PHYSICS TO OFTEN BE MORE CONVENIENT TO USE THIS R AND L NOTATION.

In more conventional 4-component Dirac notation, the left- and right-handed 2-spinors are equivalent to projections of the form

$$\begin{aligned}\psi_L &\leftrightarrow P_L\psi\\\psi_R &\leftrightarrow P_R\psi\end{aligned} \tag{14}$$

where γ_5 is a 4x4 gamma matrix and ψ is a 4-component Dirac spinor. Also $P_L = \frac{1}{2}(1 - \gamma_5)$ and $P_R = \frac{1}{2}(1 + \gamma_5)$. These projections are known as **chirality** projections.

Here is an example. In the Weyl representation of gamma matrices

$$\gamma^5 = \begin{pmatrix}-\mathbf{I} & 0\\0 & \mathbf{I}\end{pmatrix} \tag{15}$$

so that

$$P_R = \begin{pmatrix} 0 & 0 \\ 0 & \mathbf{I} \end{pmatrix}, \quad P_L = \begin{pmatrix} \mathbf{I} & 0 \\ 0 & 0 \end{pmatrix}. \tag{16}$$

Note that each matrix entry is a 2 x 2 matrix. Then

$$P_R \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = \begin{pmatrix} 0 \\ \psi_R \end{pmatrix}, \quad P_L \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = \begin{pmatrix} \psi_L \\ 0 \end{pmatrix}. \tag{17}$$

Although the vectors all have 4 components, the projected vectors can be naturally identified with 2-component spinors.

2.3 Parity

So far, we've only discussed rotations and boosts. But most of our experience with natural laws, tells us we can't distinguish left from right. That's called *parity invariance.* The transformation that accomplishes this, must take (x,y,z) to $-$(x,y,z), while leaving the t-component alone. There's a simple group property. Do a parity-transform twice, and you end up where you started. So we describe the transformation under parity as, for example,

$$\psi'_L(t', x', y', z') = \mathbf{P}\psi_L(t, -x, -y, -z) \tag{18}$$

where $\mathbf{P}^2 = \mathbf{I}$.

In 4 spacetime dimensions, the parity transformation is distinct from all rotations and boosts, so it is theoretically possible for a theory to be Lorentz invariant but not parity invariant. In fact, that happens with weak interactions.

In 2 spacetime dimensions, there is no distinct parity operation, but for simplicity I'll pretend there is so that we illustrate the issue of interest. Start with the right-handed Weyl equation

$$\begin{pmatrix} (i\partial_t + i\partial_z)\psi_R^1(x) \\ (i\partial_t - i\partial_z)\psi_R^2(x) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{19}$$

and let $z \to -z$. Then $\partial_z \to -\partial_z$ so we have the transformed equation

$$\begin{pmatrix} (i\partial_t - i\partial_z)\psi_R^1(x) \\ (i\partial_t + i\partial_z)\psi_R^2(x) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \tag{20}$$

This doesn't look the same as the original equation – although in point of fact, we could fix this by requiring the first and second components of ψ_R to be exchanged under a parity transformation. It turns out in 3 spacial dimensions that this type of transformation can't be found.

Again, for 4-component Dirac fields we can have the parity operator exchange left- and right-handed spinors and thus restore parity invariance.

2.4 Majorana masses and one of the unknowns of the standard model

We tried to add a mass term to the two-component Weyl equation and discovered that this term broke Lorentz invariance. We concluded – incorrectly – that the only way to introduce a mass term was to promote the Weyl equation into a full-blown 4-component Dirac equation.

There's another way. Consider the equation

$$\begin{pmatrix} (i\partial_t + i\partial_z)\psi_R^1(x) \\ (i\partial_t - i\partial_z)\psi_R^2(x) \end{pmatrix} = m \begin{pmatrix} \psi_R^{*2}(x) \\ \psi_R^{*1}(x) \end{pmatrix} \tag{21}$$

where we've complex-conjugated the RHS and also exchanged the two components. When we apply the boost transformation, this becomes

$$\begin{pmatrix} e^{\frac{\beta}{2}} & 0 \\ 0 & e^{-\frac{\beta}{2}} \end{pmatrix} \begin{pmatrix} (i\partial_{t'} + i\partial_{z'})\,\psi_R'^1(\mathbf{x}') \\ (i\partial_{t'} - i\partial_{z'})\psi_R'^1(\mathbf{x}') \end{pmatrix} = \begin{pmatrix} e^{\frac{\beta}{2}} & 0 \\ 0 & e^{-\frac{\beta}{2}} \end{pmatrix} m \begin{pmatrix} \psi_R'^{*2}(x') \\ \psi_R'^{*1}(x') \end{pmatrix}. \tag{22}$$

We can divide both sides by the matrix

$$\begin{pmatrix} e^{\frac{\beta}{2}} & 0 \\ 0 & e^{-\frac{\beta}{2}} \end{pmatrix} \tag{23}$$

and end up with the same form of equation we started with. So we've shown a 2-component equation which is boost invariant. We didn't actually use the fact that the RHS was conjugated.

However, if we also were to check rotation invariance, it would turn out that the conjugation is required (the boost matrix is replaced by a matrix whose components are complex).

This mass term is known as a Majorana mass. Neutrinos, which are so light they were once thought to be massless, has some characteristics that suggest it is a 2-component particle with a Majorana mass. If so, we would refer to this as a *Majorana neutrino*. To date, we don't know if neutrinos are Majorana particles.

3 The Fermion sector of the electroweak interactions

... (which also happen to be the same for each generation). Putting everything together, the gauge interactions are

$$\mathcal{L} = i\bar{L}_i(\not{\partial} - ig\not{W}^a\tau^a - ig'Y_L\not{B})L_i + i\bar{Q}_i(\not{\partial} - ig\not{W}^a\tau^a - ig'Y_Q\not{B})Q_i \\ + i\bar{e}^i_R(\not{\partial} - ig'Y_e\not{B})e^i_R + i\bar{\nu}^i_R(\not{\partial} - ig'Y_\nu\not{B})\nu^i_R \\ + i\bar{u}^i_R(\not{\partial} - ig'Y_u\not{B})u^i_R + i\bar{d}^i_R(\not{\partial} - ig'Y_d\not{B})d^i_R. \quad (29.35)$$

Notation:

- $\bar{\psi} \equiv \psi\gamma_0$ where the gamma matrices are the 4x4 Dirac matrices
- $\not{A} \equiv \sum_{\mu=0}^{3} A_\mu\gamma^\mu$
- Similarly $\not{\partial} \equiv \sum_{\mu=0}^{3} \partial_\mu\gamma^\mu$
- L_i is a left-handed lepton pair such as $\begin{pmatrix}\nu_L \\ e_L\end{pmatrix}$ where the entries are each 'chiral' fields (by which I mean $P_L\psi$ where ψ is a 4-spinor field). The subscript i denotes the generation (related to flavor).
- Q_I is a left-handed quark pair such as $\begin{pmatrix}u_L \\ d_L\end{pmatrix}$
- e_R, ν_R, u_R, d_R are all right-handed fields (i.e., right-handed projections). The first two are leptons and the last two are quarks.

- B and W^3 are neutral electro-weak vector bosons that are linear combinations of the more familiar neutral bosons A and Z. Specifically,

$$\begin{aligned} B_\mu &= \cos\theta_w A_\mu - \sin\theta_w Z_\mu \\ W^3_\mu &= \sin\theta_w A_\mu + \cos\theta_w Z_\mu \end{aligned} \tag{24}$$

where the Weinberg angle θ_W is defined by

$$\tan\theta_w = \frac{g'}{g} \tag{25}$$

- Y_a are numbers known as the hypercharge, and which, like charge, characterize the particles's $U(1)$ representation.

Table 29.1 Charges of Standard Model fields. □ indicates that the field transforms in the fundamental representation, and – indicates that a field is uncharged.

Field	$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	e_R	ν_R	$Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	u_R	d_R	H
$SU(3)$	–	–	–	□	□	□	–
$SU(2)$	□	–	–	□	–	–	□
$U(1)_Y$	$-\frac{1}{2}$	-1	0	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{1}{2}$

This is all very messy. Let's look at just at terms involving the neutral bosons W^3 and B.

$$\mathcal{L} = i\bar{L}_1 \left(\not\partial - ig\not W^3\tau^3 - ig'Y_L\not B\right) L_1 + i\bar{e}^1_R \left(\not\partial - ig'Y_e\not B\right) e^1_R + i\bar{\nu}^1_R \left(\not\partial - ig'Y_v\not B\right) \nu^1_R + ... \tag{26}$$

$L_1 = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$. Both e_L and ν_L are left-handed spinor fields. You might wonder what L_2 is. There, you replace the electron and electron-neutrino, by the muon and muon-neutrino.

$$\tau^3 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{27}$$

In experiments, we end up measuring the particles associated with linear combinations of the fields W^3 and B – namely, A and Z using the defining equations above

$$\begin{aligned} B_\mu &= \cos\theta_w A_\mu - \sin\theta_w Z_\mu \\ W^3_\mu &= \sin\theta_w A_\mu + \cos\theta_w Z_\mu \end{aligned} \tag{28}$$

with the Weinberg angle θ_W is defined by

$$\tan\theta_w = \frac{g'}{g}. \tag{29}$$

So if we expand, making the transformation to the photon (A) field and Z field we get

$$\begin{aligned} \mathcal{L} = {} & i\bar{e}_L(\not{\partial} + ig'\not{A})e_L + i\bar{e}_R(\not{\partial} + ig'\not{A})e_R - \\ & i\bar{e}_L(\frac{g'^2 - g^2}{2\sqrt{g^2 + g'^2}}\not{Z})e_L + i\bar{e}_R(\frac{g'}{2\sqrt{g^2 + g'^2}}\not{Z})e_R + \\ & \bar{\nu}_{L_e}\left(\not{\partial} + ig\not{Z}\right)\nu_{L_e} \end{aligned} \tag{30}$$

There are a number of important points here.

3.1 V-A interactions

Section 11.3 of Thomson discusses the "V-A" structure of weak interactions. The idea is this: Under a parity transformation, a vector $\mathbf{V} = (v_1, v_2, v_3)$ transforms to $(-v_1, -v_2, -v_3)$. The electric field is an example of a vector. On the other hand, consider two vectors V_1 and V_2 and their cross product $W = (w_1, w_2, w_3) = V_1 \times V_2$. Under a parity transformation, W transforms to itself, i.e. (w_1, w_2, w_3). We call W an "axial vector". An example is the magnetic field B.

Roughly speaking, "V" interactions preserve parity and "A" interactions don't.

In the language of 4-fermions, an interaction that involves the gamma matrix γ_5 is typically indicative of an axial term and thus a parity-breaking term. The γ_5 matrix is involved in the chiral projection operators P_L and P_R which project out the left-handed and right-handed fermions.

- In Eq. (30) the electron left-handed and right-handed derivative and photon interactions are identical for left and right. These are "V" types of term and are parity-conserving.
- Furthermore, because the left- and right- handed terms are identical, these can be combined into the familiar expression involving a single 4-component Dirac fermion.
- In Eq. (30) the Z-boson interaction is different for the left and right electrons, hence there will be parity-violation.
- Only the left-handed neutrinos appear.

3.2 SU(2) invariance

- All the terms are SU(2)-invariant. The right-handed fermions are SU(2)-singlets (i.e., scalars under an SU(2)-transformation). The left-handed fermions are in an SU(2)-doublet. So if you transform the left-handed neutrino and electron fields as

$$\begin{aligned} e'_L &= \cos\theta e_L + \sin\theta \nu_L \\ \nu'_L &= -\sin\theta e_L + \cos\theta \nu_L \\ e'_R &= e_R \\ \nu'_R &= \nu_R \end{aligned} \tag{31}$$

you'll get – for the noninteracting terms (i.e. the kinetic terms)

$$\mathcal{L} = i\bar{e'}_L \partial\!\!\!/ e'_L + i\bar{\nu'}_{e'L} \partial\!\!\!/ \nu'_{e'L} + i\bar{e'}_R \partial\!\!\!/ e'_R + i\bar{\nu'}_{eR} \partial\!\!\!/ \nu'_{eR} - ... \tag{32}$$

The interacting term is suppressed for now, because we haven't discussed the transformation of W.

- We don't see a Dirac mass term for the electron or neutrino. That's because such a term would violate SU(2)-invariance. We'd need some term like $m\bar{e}_L e_R$. But this would transform to

$$m\left(\cos\theta e'_L + \sin\theta \nu'_L\right) e'_R \tag{33}$$

Of course we would need to check other terms but it turns out that no combination of quadratic terms is SU(2)-invariant. **THEREFORE SU(2)-INVARIANCE PREVENTS FERMIONS FROM HAVING DIRAC MASSES**.

4 The Boson Sector

When we studied the strong interactions we concluded that the local SU(3) invariance led to a Lagrangian that looked like the following:

The boson sector of this theory is comprised of the the terms on the first and third lines of the RHS. The form of the Lagrangian looks fairly simple partly because the gluon fields are all indexed – e.g. A^{μ}_{a}. By contrast, the SU(2) x U(1) Lagrangian terms look considerably more complicated, largely because each vector boson is separately identified.

$$\begin{aligned}
\mathcal{L}_{\text{gauge}} = & -\frac{1}{4}F^2_{\mu\nu} - \frac{1}{4}Z^2_{\mu\nu} + \frac{1}{2}m_Z^2 Z^{\mu} Z_{\mu} - \frac{1}{2}(\partial_{\mu}W^+_{\nu} - \partial_{\nu}W^+_{\mu})(\partial_{\mu}W^-_{\nu} - \partial_{\nu}W^-_{\mu}) \\
& + m_W^2 W^+_{\mu}W^-_{\mu} - ie\cot\theta_w \Big[\partial_{\mu}Z_{\nu}(W^+_{\mu}W^-_{\nu} - W^+_{\nu}W^-_{\mu}) \\
& \qquad + Z_{\nu}(-W^+_{\mu}\partial_{\nu}W^-_{\mu} + W^-_{\mu}\partial_{\nu}W^+_{\mu} + W^+_{\mu}\partial_{\mu}W^-_{\nu} - W^-_{\mu}\partial_{\mu}W^+_{\nu}) \Big] \\
& - ie\Big[\partial_{\mu}A_{\nu}(W^+_{\mu}W^-_{\nu} - W^+_{\nu}W^-_{\mu}) \\
& \qquad + A_{\nu}(-W^+_{\mu}\partial_{\nu}W^-_{\mu} + W^-_{\mu}\partial_{\nu}W^+_{\mu} + W^+_{\mu}\partial_{\mu}W^-_{\nu} - W^-_{\mu}\partial_{\mu}W^+_{\nu}) \Big] \\
& - \frac{1}{2}\frac{e^2}{\sin^2\theta_w}W^+_{\mu}W^-_{\mu}W^+_{\nu}W^-_{\nu} + \frac{1}{2}\frac{e^2}{\sin^2\theta_w}W^+_{\mu}W^-_{\nu}W^+_{\mu}W^-_{\nu} \\
& - e^2\cot^2\theta_w(Z_{\mu}W^+_{\mu}Z_{\nu}W^-_{\nu} - Z_{\mu}Z_{\mu}W^+_{\nu}W^-_{\nu}) + e^2(A_{\mu}W^+_{\mu}A_{\nu}W^-_{\nu} - A_{\mu}A_{\mu}W^+_{\nu}W^-_{\nu}) \\
& + e^2\cot\theta_w\Big[A_{\mu}W^+_{\mu}W^-_{\nu}Z_{\nu} + A_{\mu}W^-_{\mu}Z_{\nu}W^+_{\nu} - W^+_{\mu}W^-_{\mu}A_{\nu}Z_{\nu} \Big], \qquad (29.9)
\end{aligned}$$

with

hypercharge gauge groups, the Higgs boson h does not. Expanding out the Lagrangian, we find that the terms involving the Higgs boson are

$$\mathcal{L}_{Higgs} = -\frac{1}{2}h(\Box + m_h^2)h - g\frac{m_h^2}{4m_W}h^3 - \frac{g^2}{32}\frac{m_h^2}{m_W^2}h^4$$
$$+ 2\frac{h}{v}\left(m_W^2 W_\mu^+ W_\mu^- + \frac{1}{2}m_Z^2 Z_\mu^2\right) + \left(\frac{h}{v}\right)^2\left(m_W^2 W_\mu^+ W_\mu^- + \frac{1}{2}m_Z^2 Z_\mu Z_\mu\right), \qquad (29.14)$$

where $m_h = \sqrt{2}m$. Using $v = \frac{2m_W \sin\theta_w}{e}$, the Feynman rule for a Higgs boson interacting with two W bosons is

These two contributions to the Lagrangian constitute the boson sector. The vector bosons are W^+, W^-, Z, A and the Higgs (scalar) boson is h. There are several new parameters:

- v is the value by which we have to shift the Higgs field in order to expand around our vacuum of choice. v is sometimes called the *vacuum expectation value* or *VEV*
- e is the ordinary electromagnetic coupling constant
- M_Z, M_W and m_h are respectively the masses of the Z, $W^\pm$ and h with $M_Z = \frac{M_W}{\cos(\theta_W)}$

One immediate prediction of the theory is that the $W^\pm$ bosons should be lighter than the Z boson.

5 Feynman rules and the Fermi theory

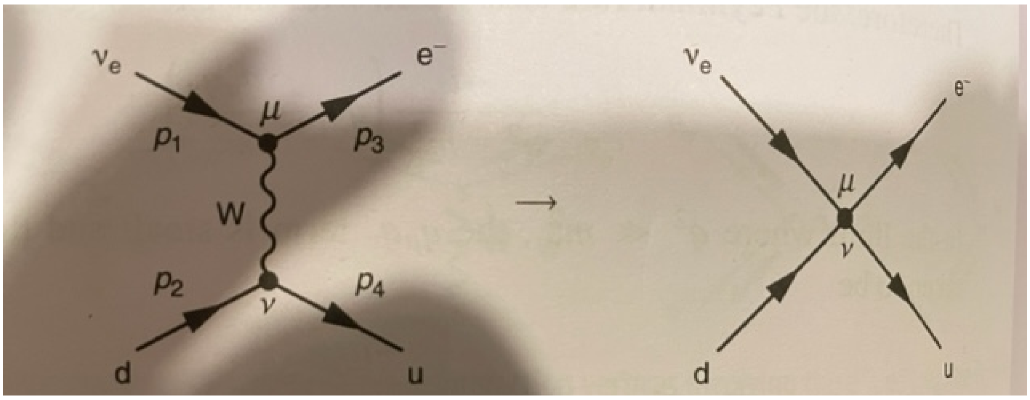
(See Thomson Section 11.5). Let's examine what the Feynman rules tell us. Recall the weak interaction Lagrangian for leptons and quarks.

each generation). Putting everything together, the gauge interactions are

$$\mathcal{L} = i\bar{L}_i(\not{\partial} - ig\not{W}^a\tau^a - ig'Y_L\not{B})L_i + i\bar{Q}_i(\not{\partial} - ig\not{W}^a\tau^a - ig'Y_Q\not{B})Q_i$$
$$+ i\bar{e}^i_R(\not{\partial} - ig'Y_e\not{B})e^i_R + i\bar{\nu}^i_R(\not{\partial} - ig'Y_\nu\not{B})\nu^i_R$$
$$+ i\bar{u}^i_R(\not{\partial} - ig'Y_u\not{B})u^i_R + i\bar{d}^i_R(\not{\partial} - ig'Y_d\not{B})d^i_R. \quad (29.35)$$

Remember that the scattering amplitudes are computed by joining vertices (the non-quadratic terms) with propagators (the quadratic terms).

For example, consider the LHS of the diagram



The top vertex comes from a term with a product of fields for ν_e, e and W. We see that the term of interest is

$$g\bar{L}_i\not{W}^a\tau^a L_i. \quad (34)$$

The detailed vertex value comes from evaluating the Dirac matrices including the chiral projection operator (which projects the 2 left-hand components of the leptons) and the SU(2) Pauli matrices (τ).

Similarly the bottom vertex comes from a product of W with the two quark fields b and u, which form the constituents of – for example – a proton, corresponding to the Lagrangian term

$$g\bar{Q}_i \not{W}^a \tau^a Q_i. \tag{35}$$

The vertices are joined by the W propagator, whose value is proportional to

$$\frac{1}{q^2 - m_W^2} \tag{36}$$

where $q = p_1 - p_3$ and m_W is the mass of the W-boson (80.4 GeV). Then the value of the scattering amplitude (which doesn't take into account the details of incoming and outgoing momentum density profiles) is roughly

$$M = g^2 \frac{V_1 \times V_2}{q^2 - m_W^2} \tag{37}$$

where V_1 and V_2 are the two vertices (modulo the coupling constants)

This diagram would represent, for instance, a collision of a neutrino with a neutron, resulting in an electron and a proton (if you convert a neutron's down quark into an up quark, you get a proton). In most 'ordinary' collisions (e.g. old-fashioned labs, irradiation, atomic bombs ...) the typical energies are very small so $|q| << 80.4GeV$.

In such a case, the W-propagator is approximately -0.00015GeV^{-2} and the amplitude is approximately independent of the momenta. We can then rewrite the amplitude in a form which, by convention, looks like

$$M = \frac{G_F}{\sqrt{2}} V_{12} \tag{38}$$

where G_F is the so-called "Fermi constant" and has the value $G_F = 1.16638 \times 10^{-5}\text{GeV}^{-2}$.

Until the 1960's, physicists had worked backwards from experiment, to conclude that the weak interaction Lagrangian was actually a product of 4 fermion fields as shown on the RHS of the above diagram and with an interaction strength given by the Fermi constant. They concluded this because of the momentum-independence of the amplitude. That theory was known as the "Fermi theory". It was a momentous triumph of intellect to guess that the correct theory involved the exchange of a very massive particle (the W boson) and furthermore, that this particle acquired its mass through the Higgs mechanism. The discovery of the Higgs boson ultimately was the last frontier for confirmation of the entire framework responsible for the Fermi interaction.

One reason that 'renormalizability' was 'de rigeur' in the 1970's and later, is that the death knell of the Fermi theory started when field theorists realized that the Fermi Lagrangian was non-renormalizable. By demanding renormalizability, physicists eventually were led to the current standard model. This was such a resounding success that until recently, renormalizability was regarded as sacred. That viewpoint has shifted a bit, but with some revisionist history pointing out that Fermi theory had been remarkably successful for several decades notwithstanding its non-renormalizability.