# Neutrino oscillations

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## 1 Summary of the weak interactions of fermions

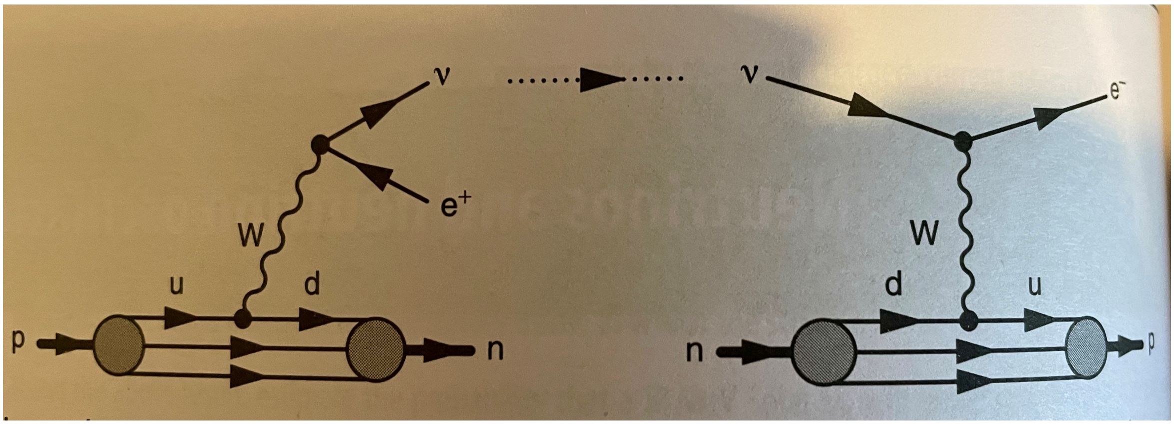
Recall the weak interaction Lagrangian for leptons and quarks.

... hypercharges (which also happen to be the same for each generation). Putting everything together, the gauge interactions are

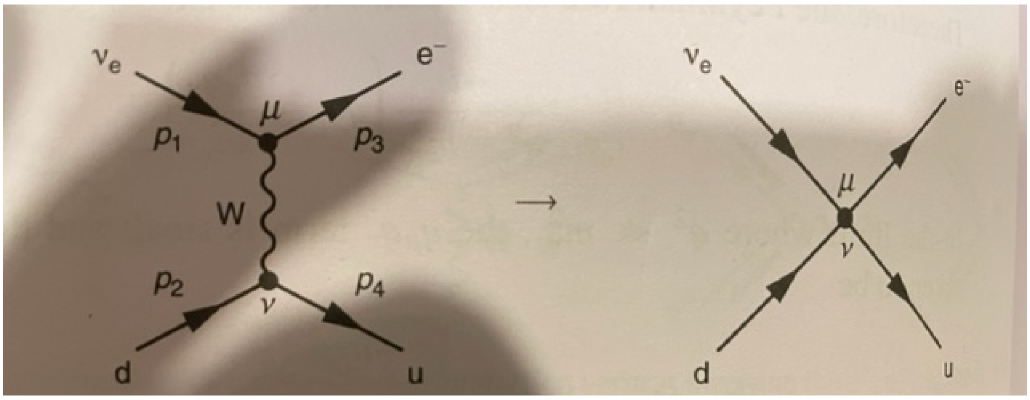
$$\mathcal{L} = i\bar{L}_i(\partial\!\!\!/ - igW\!\!\!\!/^a\tau^a - ig'Y_L B\!\!\!/)L_i + i\bar{Q}_i(\partial\!\!\!/ - igW\!\!\!\!/^a\tau^a - ig'Y_Q B\!\!\!/)Q_i$$
$$+ i\bar{e}^i_R(\partial\!\!\!/ - ig'Y_e B\!\!\!/)e^i_R + i\bar{\nu}^i_R(\partial\!\!\!/ - ig'Y_\nu B\!\!\!/)\nu^i_R$$
$$+ i\bar{u}^i_R(\partial\!\!\!/ - ig'Y_u B\!\!\!/)u^i_R + i\bar{d}^i_R(\partial\!\!\!/ - ig'Y_d B\!\!\!/)d^i_R. \quad (29.35)$$

Remember that the scattering amplitudes are computed by joining vertices (the non-quadratic terms) with propagators (the quadratic terms).

Some characteristic Feynman diagrams are shown here.




Consider the diagram on the right and simplify it to the following diagram on the left.




The top vertex comes from a term with a product of fields for $\nu_e$, $e$ and $W$. We see that the term of interest is

$$g\bar{L}_i \not{W}^a \tau^a L_i. \tag{1}$$

Similarly the bottom vertex comes from a product of W with the two quark fields $b$ and $u$, which form the constituents of – for example – a proton, corresponding to the Lagrangian term

$$g\bar{Q}_i \not{W}^a \tau^a Q_i. \tag{2}$$

The vertices are joined by the $W$ propagator, whose value is proportional to

$$\frac{1}{q^2 - m_W^2} \tag{3}$$

where $q = p_1 - p_3$ and $m_W$ is the mass of the $W$-boson (80.4 GeV). Then the value of the scattering amplitude (which doesn't take into account the details of incoming and outgoing momentum density profiles) is roughly

$$M = g^2 \frac{V_1 \times V_2}{q^2 - m_W^2} \tag{4}$$

where $V_1$ and $V_2$ are the two vertices (modulo the coupling constants)

This diagram would represent, for instance, a collision of a neutrino with a neutron, resulting in an electron and a proton (if you convert a neutron's down quark into an up quark, you get a proton). In most 'ordinary' collisions (e.g. old-fashioned labs, irradiation, atomic bombs ...) the typical energies are very small so $|q| << 80.4GeV$.

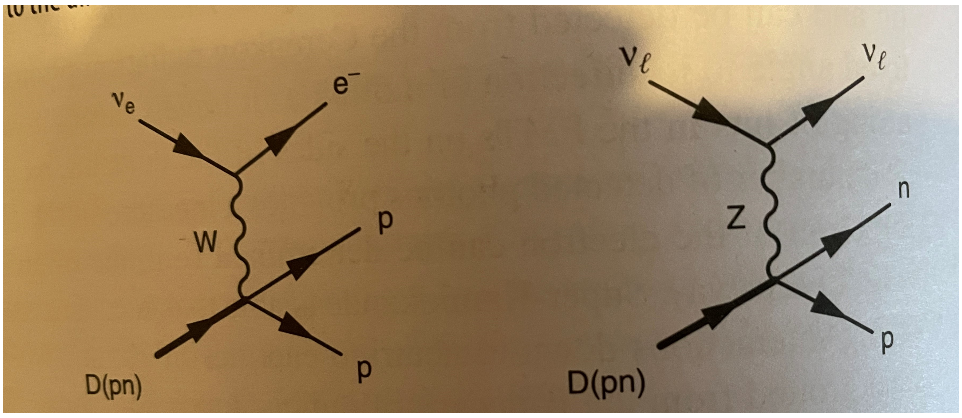
**SO FAR, WE HAVEN'T INCLUDED INTERACTIONS WITH THE HIGGS BOSON.**

# 2 Solar neutrinos

(Thomson Chapter 13)

Nuclear fusion processes are well understood, and lead to the production of neutrinos and other particles. The other particles get absorbed on their way to the earth, but since neutrinos interact weakly, those directed toward the earth mostly aren't deflected or absorbed before hitting the earth's surface. Neutrino detectors consist of massive amounts of special dense materials (e.g. dry-cleaning fluid). The first statistically significant 'smoking gun' experiment, called the Homestake Mine experiment, observed about 0.5 neutrino interactions per day. However, solar-reaction calculations predicted about 1.7 neutrino interactions per day. The best explanation – one which was subsequently corroborated by some accurate experiments – was that the neutrinos had changed character on the way towards earth.

To understand how this happens, consider what happens when an electron neutrino – the only kind of neutrino that can be produced (with any abundance) in the sun – encounters a deuteron.




In this diagram, there are two processes shown. On the left, the electron neutrino is 'converted' to an electron through the exchange of the charged vector boson $W^\mu$, while the deuteron is converted to a pair of protons. On the right, the electron neutrino is deflected (into another electron neutrino) and the deuteron is converted into its component neutron and proton. The original Homestead experiment only focused on the reactions of the type on the left.

**N**otice the right diagram also shows that muon and tau neutrinos can interact with the deuteron. This turns out to be critical! It was originally thought that neutrinos are massless, and it can be shown as we'll see below that in that case, the only solar neutrinos captured on earth must be the ones produced on the sun – namely electron neutrinos.

On the other hand, if neutrinos have mass, then electron neutrinos can turn into muon or tau neutrinos and those would evade detection from the diagram on the left. On the other hand, they can be measured by using the diagram on the right – as was done by the Sudbury Neutrino Observatory. The punch line is that once these are included, the total number of neutrinos captured are in agreement with the prediction for the production of neutrinos by the sun.

## 3 The neutrino mass matrix

Up to now, we've ignored interactions between the Higgs boson and leptons – in particular neutrinos. It's permissible to add a Higgs-lepton interaction term to the Lagrangian, that respects the SU(2) x U(1) electroweak symmetry.

$$\mathcal{L}_{\text{mass}} = -Y^e_{ij}\bar{L}^i H e^j_R - Y^\nu_{ij}\bar{L}^i(i\sigma_2 H^*)\nu^j_R + h.c. + ... \tag{5}$$

There are two key observations to be made:

- The indices $i$ and $j$ denote flavor and so these terms mix flavors (e.g. electron and muon neutrinos)
- Although $H$ is the Higgs boson field, the lowest order expansion of the path integral is done around a non-zero value of $H$ – the 'vacuum expectation value' – which then turns the above Lagrangian into a quadratic term.

So let's simplify this argument. (Thomson does a nice job of that.) What we've ended up with, is a set of Feynman diagrams whose vertices come from the interaction terms in the Lagrangian and in particular **represent interactions between *fields***. We tend to refer to $\nu_e$ as an electron neutrino 'particle'. But that's not strictly correct. It really is the electron neutrino 'field'.

Ignore the tau neutrino. Write the electron and muon neutrino fields as

$$\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} \tag{6}$$

Here, $\nu_1$ and $\nu_2$ are fields which diagonalize the mass matrix. Those are the appropriate fields which correspond to particles.

In summary, the fields that appear in the standard Lagrangian are the ones which participate in interactions, but the particles that we observe are ones of definite mass (i.e mass eigenvalues) and correspond to fields that are linear superpositions of the 'standard' fields.

Now let's consider particle states with momentum and energy

$$\begin{aligned}|\nu_1(t)\rangle &= |\nu_1\rangle e^{i(\mathbf{p}_1\cdot\mathbf{x}-E_1 t)}\\ |\nu_2(t)\rangle &= |\nu_2\rangle e^{i(\mathbf{p}_2\cdot\mathbf{x}-E_2 t)}\end{aligned} \tag{7}$$

Now suppose at time $t = 0$, an electron neutrino is produced in a solar process. The initial wavefunction is

$$|\psi(0)\rangle = |\nu_e\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle \tag{8}$$

The state evolves. A proper treatment of all this, should be done using wave packets. However, the key results can be obtained by the following argument. Assume that the neutrino state propagates at the speed of light, so that it travels a distance $L = cT$ in time $T$. Set $c = 1$ for notational simplicity, pretend the whole thing is in one spatial dimension and obtain

$$|\psi(L,T)\rangle = \cos\theta|\nu_1\rangle e^{-i(E_1T-p_1L)} + \sin\theta|\nu_2\rangle e^{-i(E2T-p21L)} \tag{9}$$

Now write $|\nu_1\rangle$ and $|\nu_2\rangle$ in terms of $|\nu_e\rangle$ and $|\nu_\mu\rangle$ using the inverse of Eq. (6) leading to Thomson eq. (13.8)

$$|\psi(L,T)\rangle = e^{-i\phi_1}\left[\left(\cos^2\theta + e^{i\Delta\phi_{12}}\sin^2\theta\right)|\nu_e\rangle - \left(1 - e^{-i\Delta\phi_{12}}\right)\cos\theta\sin\theta|\nu_\mu\rangle\right] \tag{10}$$

where we've defined $\phi_i = E_iT = p_iL$ and $\phi_{12} = \phi_1 - \phi_2$. When we take the magnitude of the state, we see a familiar oscillation term (beat frequency) if $\Delta\phi_{12} \neq 0$.

Thomson does a crude analysis showing that

$$\Delta\phi_{12} \approx \frac{m_1^2 - m_2^2}{2p}L \tag{11}$$

and then illustrates this with the graph