

Jordan Normal Forms of 4×4 Matrices: All Possible Eigenvalue Multiplicities

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Non-Defective	Defective		
$\begin{matrix} \{1111\} \\ \{1111\} \\ \left[\begin{array}{cccc} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 \\ 0 & 0 & \lambda_3 & 0 \\ 0 & 0 & 0 & \lambda_4 \end{array} \right] \end{matrix}$			
$\begin{matrix} \{211\} \\ \{211\} \\ \left[\begin{array}{cccc} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_1 & 0 & 0 \\ 0 & 0 & \lambda_2 & 0 \\ 0 & 0 & 0 & \lambda_3 \end{array} \right] \end{matrix}$	$\begin{matrix} \{211\} \\ \{111\} \\ \left[\begin{array}{cccc} \lambda_1 & 1 & 0 & 0 \\ 0 & \lambda_1 & 0 & 0 \\ 0 & 0 & \lambda_2 & 0 \\ 0 & 0 & 0 & \lambda_3 \end{array} \right] \end{matrix}$		
$\begin{matrix} \{22\} \\ \{22\} \\ \left[\begin{array}{cccc} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_1 & 0 & 0 \\ 0 & 0 & \lambda_2 & 0 \\ 0 & 0 & 0 & \lambda_2 \end{array} \right] \end{matrix}$	$\begin{matrix} \{22\} \\ \{12\} \\ \left[\begin{array}{cccc} \lambda_1 & 1 & 0 & 0 \\ 0 & \lambda_1 & 0 & 0 \\ 0 & 0 & \lambda_2 & 0 \\ 0 & 0 & 0 & \lambda_2 \end{array} \right] \end{matrix}$	$\begin{matrix} \{22\} \\ \{11\} \\ \left[\begin{array}{cccc} \lambda_1 & 1 & 0 & 0 \\ 0 & \lambda_1 & 0 & 0 \\ 0 & 0 & \lambda_2 & 1 \\ 0 & 0 & 0 & \lambda_2 \end{array} \right] \end{matrix}$	
$\begin{matrix} \{31\} \\ \{31\} \\ \left[\begin{array}{cccc} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_1 & 0 & 0 \\ 0 & 0 & \lambda_1 & 0 \\ 0 & 0 & 0 & \lambda_2 \end{array} \right] \end{matrix}$	$\begin{matrix} \{31\} \\ \{21\} \\ \left[\begin{array}{cccc} \lambda_1 & 1 & 0 & 0 \\ 0 & \lambda_1 & 0 & 0 \\ 0 & 0 & \lambda_1 & 0 \\ 0 & 0 & 0 & \lambda_2 \end{array} \right] \end{matrix}$	$\begin{matrix} \{31\} \\ \{11\} \\ \left[\begin{array}{cccc} \lambda_1 & 1 & 0 & 0 \\ 0 & \lambda_1 & 1 & 0 \\ 0 & 0 & \lambda_1 & 0 \\ 0 & 0 & 0 & \lambda_2 \end{array} \right] \end{matrix}$	
$\begin{matrix} \{4\} \\ \{4\} \\ \left[\begin{array}{cccc} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_1 & 0 & 0 \\ 0 & 0 & \lambda_1 & 0 \\ 0 & 0 & 0 & \lambda_1 \end{array} \right] \end{matrix}$	$\begin{matrix} \{4\} \\ \{3\} \\ \left[\begin{array}{cccc} \lambda_1 & 1 & 0 & 0 \\ 0 & \lambda_1 & 0 & 0 \\ 0 & 0 & \lambda_1 & 0 \\ 0 & 0 & 0 & \lambda_1 \end{array} \right] \end{matrix}$	$\begin{matrix} \{4\} \\ \{2\} \\ \left[\begin{array}{cccc} \lambda_1 & 1 & 0 & 0 \\ 0 & \lambda_1 & 1 & 0 \\ 0 & 0 & \lambda_1 & 0 \\ 0 & 0 & 0 & \lambda_1 \end{array} \right] \end{matrix}$	$\begin{matrix} \{4\} \\ \{1\} \\ \left[\begin{array}{cccc} \lambda_1 & 1 & 0 & 0 \\ 0 & \lambda_1 & 1 & 0 \\ 0 & 0 & \lambda_1 & 1 \\ 0 & 0 & 0 & \lambda_1 \end{array} \right] \end{matrix}$