Experimental violation of Bell inequalities

Based on Alain Aspect’s
“BELL’S THEOREM : THE NAIVE VIEW OF AN EXPERIMENTALIST”

(“Quantum [Un]speakables – From Bell to Quantum information”, edited by R. A. Bertlmann and A. Zeilinger, Springer (2002).)

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Review

- 1900 – 1960's: Early developments of QM.
  - Mathematical formalism and infrastructure by Einstein, Heisenberg, Schrodinger, Dirac etc.
  - "Foundations of QM": QM derived from reasonable axioms
    - von Neumann, Segal, Mackey, etc. – axiom(s) of *quantum logic* leading to Heisenberg's Hilbert space method
    - Einstein, Schrodinger, Bohm, etc. – quantum logic is NOT reasonable because it implies a *non-realistic* theory
  - By 1960's, "foundations of QM" was a fringe part of QM, regarded as purely philosophical
    - All that changed starting in 1964 with the publication of Bell's theorem

(Alain Aspect, reminiscing in 2000 about planning his Nobel-prize winning experiment) I got an appointment *(with John Bell)* … and I showed up in John's office at CERN…. While I was explaining my planned experiment, he silently listened. Eventually, I stopped talking, and the first question came: "Have you a permanent position?" … his answer reminds me of the general atmosphere at that time about raising questions on the foundations of quantum mechanics. Quite frequently it was open hostility, and in the best case, it would provoke an ironical reaction: "Quantum Mechanics has been vindicated by such a large amount of work by the smartest theorists and experimentalists, how can you hope to find anything with such a simple scheme, in optics, a science of the XIXth century?"

- 1960's – present : Quantum engineering
  - Bell used to jokingly tell people that his area of expertise was *quantum engineering.* That's now a field.
  - Experimental exposure of quantum entanglement:
    - Experiments starting in 1970's by Clauser, Aspect, Zeilinger and collaborators confirmed *non-realism*
    - Entangled systems are now being used in fields like quantum computation, cryptography, etc.

Reality

- In classical European philosophy, *reality* is a notion deriving from Plato
  - Plato speaks of people imprisoned in a cave who can only see shadows cast by the world outside the cave.
  - Plato's metaphor is that we are like the prisoners who can only see shadows. **REALITY** is the world casting shadows.
  - *(from Alfred North Whitehead Process and Reality)* "The safest general characterization of the European philosophical tradition is that it consists of a series of footnotes to Plato."
- Einstein (EPR) argued that QM wasn't a ***complete*** description of reality.
- Einstein's statement sounds philosophical, but Bohm and then Bell etc. gave it a mathematical meaning:
  - A ***complete*** description of reality requires that all experimental observations can be derived from a set of parameters (variables) (possibly distributed following traditional rules of probability and statistics.)
  - Standard QM employs *the usual* set of parameters like spin, polarization, position etc. A *complete description requires* **supplementary variables,** more often called **hidden variables.**
- Until the late 1960's **no experiment had ever been done that ruled out the possibility of such hidden variables!!!!!**
- In fact, until Bell's theorem, no-one realized that an experiment could be constructed to rule out **any** hidden variable theory.
- The experiments of Aspect and others established conclusively that ***nature is inconsistent with Plato's reality***.
- …which leaves us with *quantum logic* as a (totally unintuitive) replacement for Plato's reality

Early experiments to test "reality"

- Clauser and Freedman, 1972
- Holt and Pipkin, 1972 (incorrect result)
- Clauser, 1976
- Fry and Thompson, 1976
- Aspect et al., 1981-2(much improved)

These use a source (often calcium or mercury) that produce two oppositely-directed photons with equal polarizations (owing to angular momentum conservation etc.)

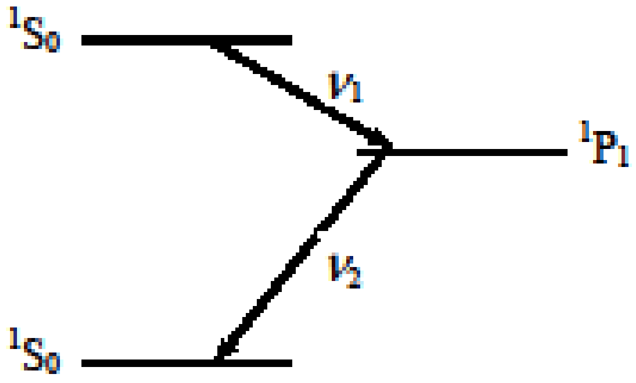


Figure 6 - Radiative cascade emitting pairs of photons correlated in polarization.

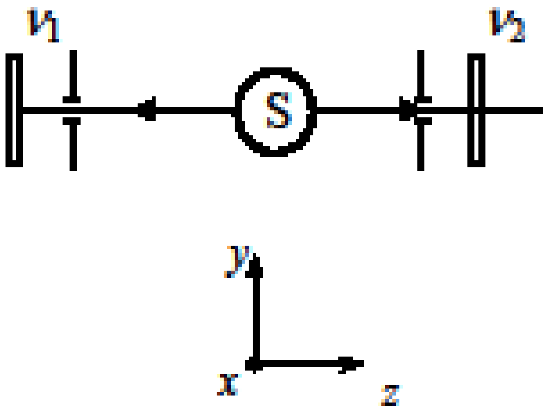
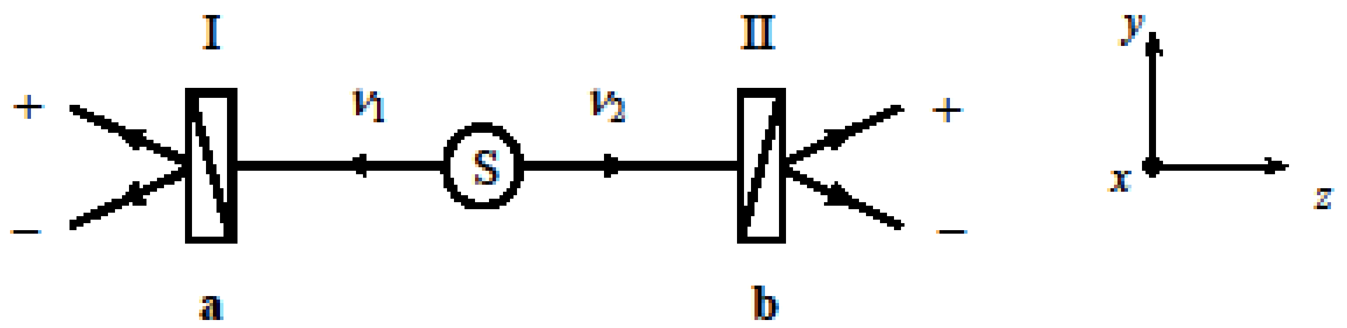
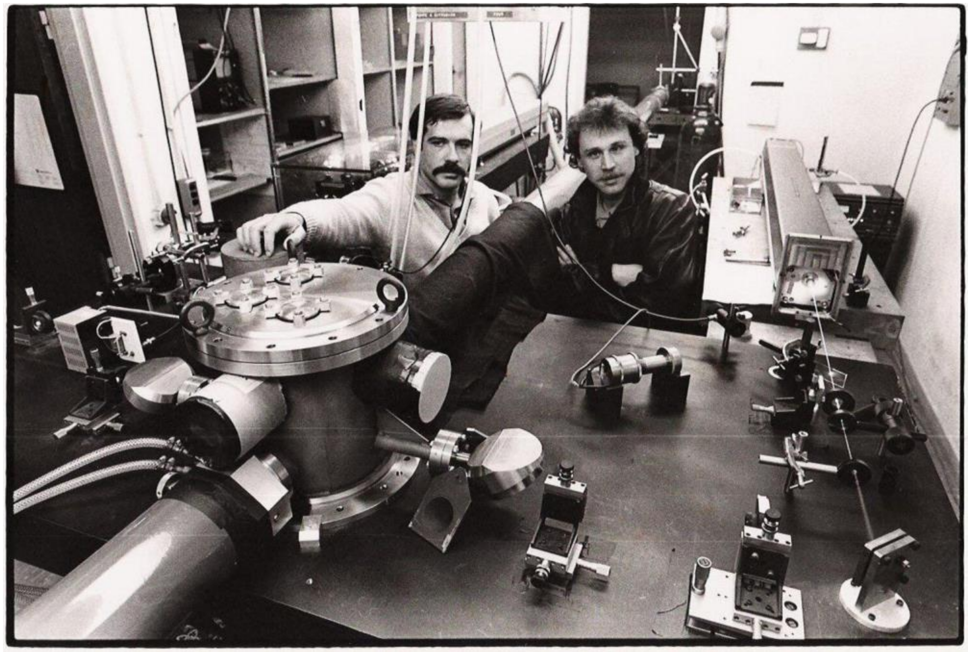


Figure 7 - Ideal configuration (infinitely small solid angles).



2-channel polarizers with orientation vectors **a** and **b** – similar to the Mermin gedanken setup



The two photons are intercepted by polarizers.

- A (standard) *one-channel polarizer* allows passage of a photon whose polarization is aligned with **a** but blocks photons with orthogonal polarization.
- A *two-channel polarizer* (e.g. built as a polarizing cube with dielectric layers) allows passage of the parallel-polarization photon and *reflects* photons with orthogonal polarization.
- The polarizer angles can be set independently.

Comparing the Bell gedanken experiment (*review)*

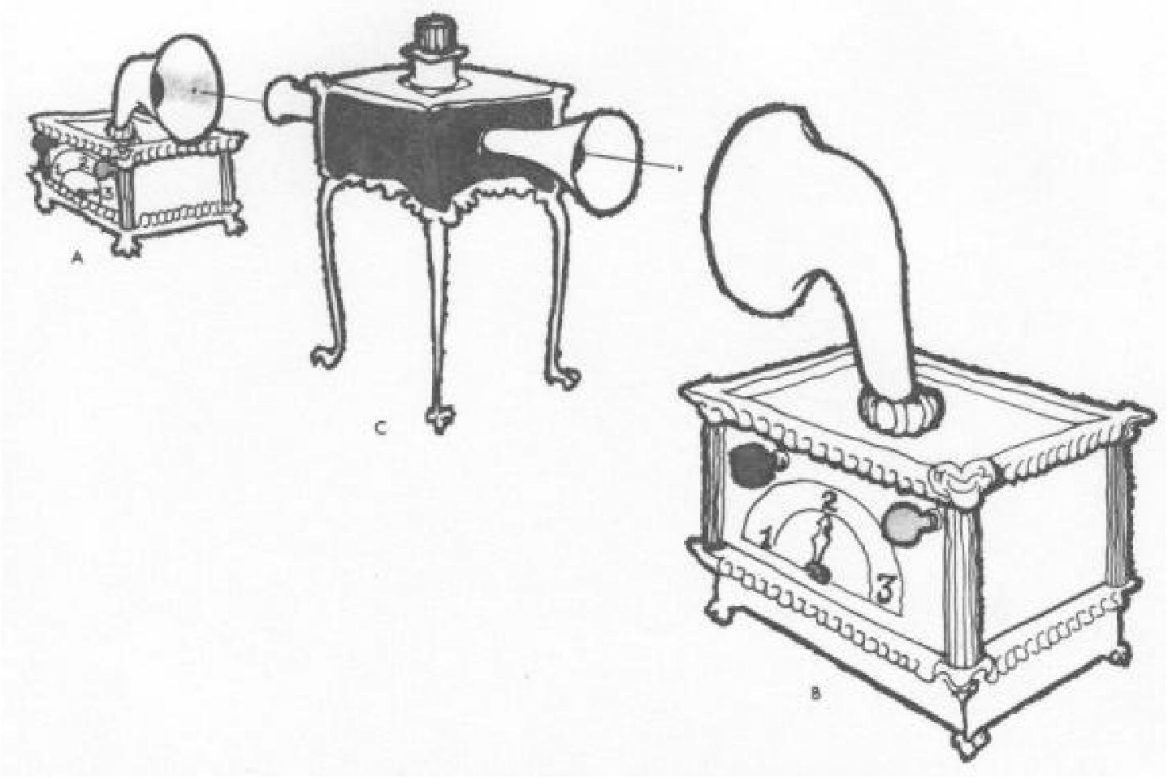
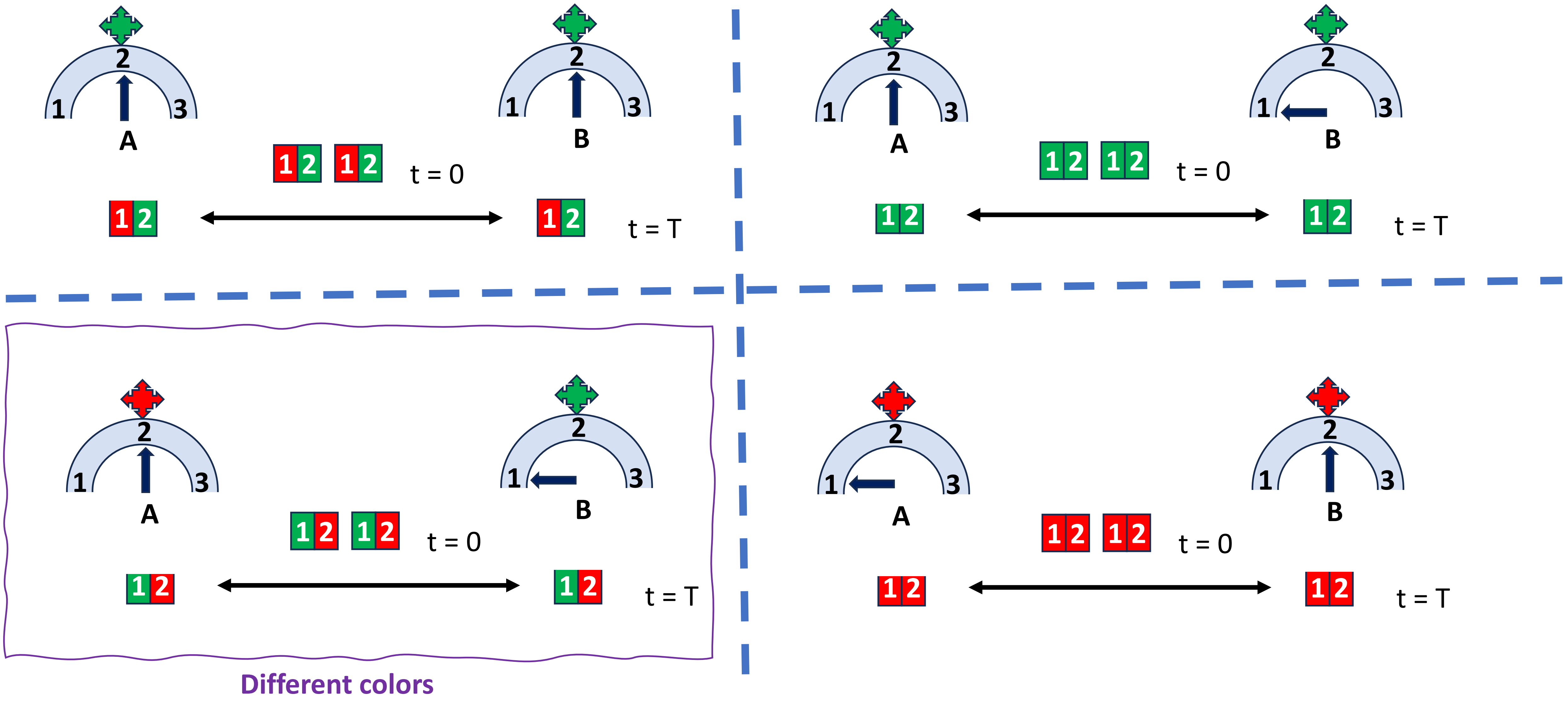


Figure 1 - An EPR apparatus.
The experimental setup consist of two detector, **A** and **B**, and a source of something ("particles" or whatever) **C**. To start a run, the experimenter pushes the button on **C**; something passes from **C** to both detectors. Shortly after the button is pushed each detector flashes one of its lights. Putting a brick between the source and one of the detectors prevents that detectors from flashing, and moving the detectors farther away from the source increases the delay between when the button is pushed and when the lights flash. The switch settings on the detectors vary randomly from one run to another. Note that there are no connections between the three parts of the apparatus, other than via whatever it is that passes from **C** to **A** and **B**.

- **When detectors are in the same position, they always flash the same color.**
- **Otherwise, they may or may not flash the same color.**
- **Reality ➡ each particle has (at least) two *hidden variables*, one per position.**



The Full Bell Chart

- All positions of detectors A and B are set for equal amounts of time
- P(Y) is the probability that both detectors flash the same color (i.e., "Y")

λ =1

1 2 3

P(Y) = 5/9

	A/1	A/2	A/3
B/1	Y	N	N
B/2	N	Y	Y
B/3	N	Y	Y

λ =2

1 2 3

P(Y) = 5/9

	A/1	A/2	A/3
B/1	Y	Y	N
B/2	Y	Y	N
B/3	N	N	Y

λ =3

1 2 3

P(Y) = 9/9

	A/1	A/2	A/3
B/1	Y	Y	Y
B/2	Y	Y	Y
B/3	Y	Y	Y

λ =4

1 2 3

P(Y) = 5/9

	A/1	A/2	A/3
B/1	Y	N	Y
B/2	N	Y	N
B/3	Y	N	Y

λ =5

1 2 3

P(Y) = 5/9

	A/1	A/2	A/3
B/1	Y	N	Y
B/2	N	Y	N
B/3	Y	N	Y

λ =6

1 2 3

P(Y) = 9/9

	A/1	A/2	A/3
B/1	Y	Y	Y
B/2	Y	Y	Y
B/3	Y	Y	Y

λ =7

1 2 3

P(Y) = 5/9

	A/1	A/2	A/3
B/1	Y	Y	N
B/2	Y	Y	N
B/3	N	N	Y

λ =8

1 2 3

P(Y) = 5/9

	A/1	A/2	A/3
B/1	Y	N	N
B/2	N	Y	Y
B/3	N	Y	Y

- λ characterizes the hidden variables.
- Model is defined by **ρ(λ), the probability of particles having λ.**

Bell's Theorem

What is the <u>overall probability</u> (**O.P.**) that both flash the same color?

- The only situations with P(Y) = 1, are when all 3 properties are the same. The rest have P(Y) = ***5/9.***
- ***Different hidden variables models (ρ) have different probabilities of the 3 properties being the same.***
- ***Suppose it never happens that 3 properties are the same. Then O.P. = 5/9***
- ***Suppose it always happens that 3 properties are the same. Then O.P. = 1***

THEREFORE, BELL'S THEOREM: FOR <u>ANY</u> HIDDEN VARIABLE MODEL (ρ), O.P.(Y) ≥ 5/9

In deriving this result, we implicitly assumed signals can't travel faster than *c*.
This is an example of *<u>locality</u>* so we speak of *<u>local hidden variable models.</u>*

Generalized Bell Theorem

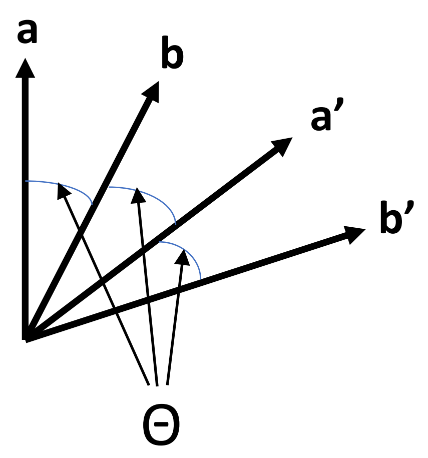
- More generally, Bell's theorem introduces probabilities P_{++}(I,J), P_{--}(I,J), P_{-+}(I,J), P_{+-}(I,J)
  - In the Aspect experiment, + and – represent final polarizations after passing through polarizers.
  - + instead of "flashes red", - instead of "flashes green"
  - The arguments (I,J) represent positions of the detector. For Aspect, these are polarizer angles (α, β).
  - The P's are probabilities of seeing the polarization-pair. These depend on the hidden-variable model

- The probability of both polarizations the same is $P_S(\alpha,\beta) = P_{++}(\alpha,\beta) + P_{--}(\alpha,\beta)$.
- The probability of both polarizations different is $P_D(\alpha,\beta) = P_{+-}(\alpha,\beta) + P_{-+}(\alpha,\beta)$.
- Define $E(a,b) = P_S(a,b) - P_D(a,b)$
- Define $S(a,a',b,b') = E(a,b) - E(a,b') + E(a',b) + E(a',b')$

In the "reality" (hidden parameters) model, P_S and P_D are computed from ρ

For any local hidden variables model, **$-2 \leq S(a,a',b,b') \leq 2$**

Aspect experiment with Θ-configuration



S(Θ) ≡ S(a,b,a',b')

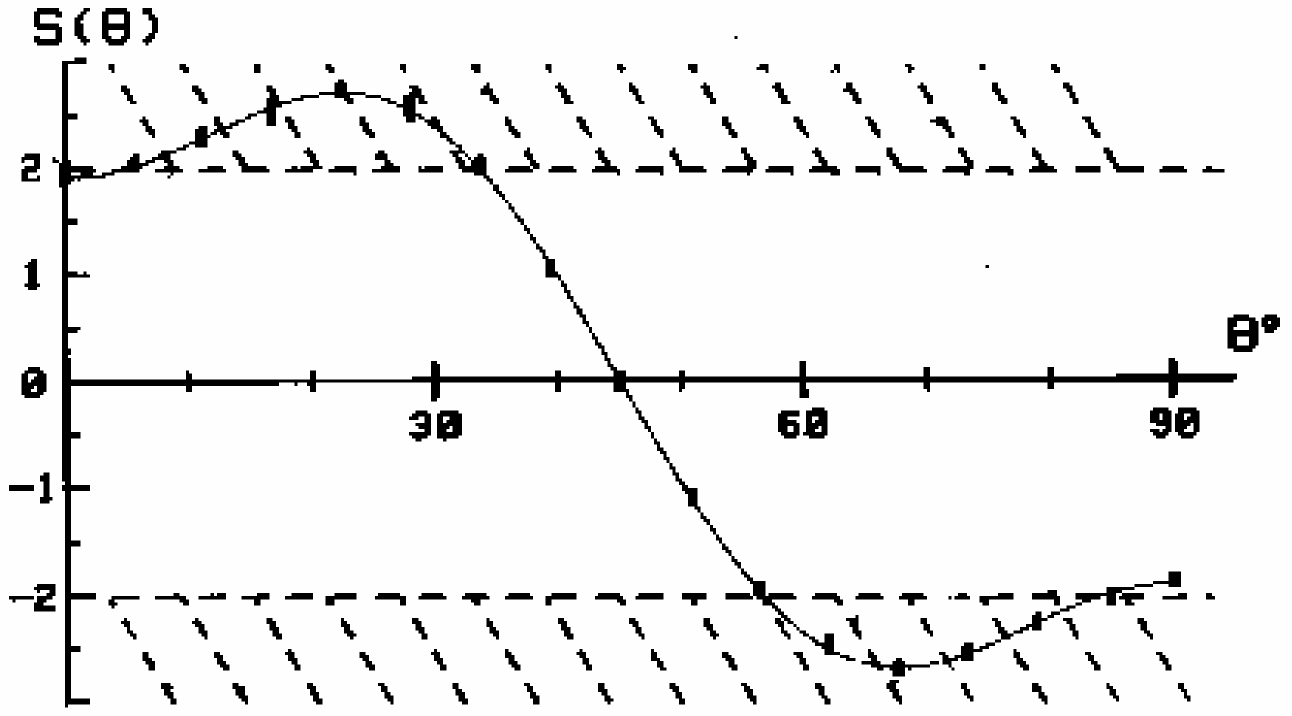
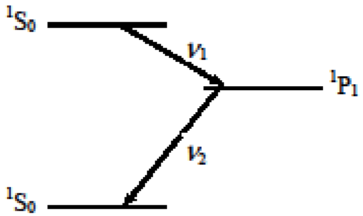


Figure 14 - Experiment with two-channels polarizers. Quantity S(θ), to be tested by Bell's inequalities ($-2 \leq S \leq +2$), as a function of the relative angle of the polarimeters. The indicated errors are ± 2 standard deviations. The dashed curve is not a fit to the data, but Quantum Mechanical predictions for the actual experiment. For an ideal experiment, the curve would exactly reach the values ± 2.828.

Quantum mechanics: the entangled state

In QM, P_S and P_D are computed from the wave function.



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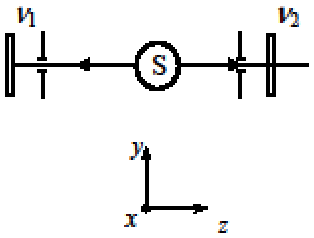


Figure 7 - Ideal configuration (infinitely small solid angles).

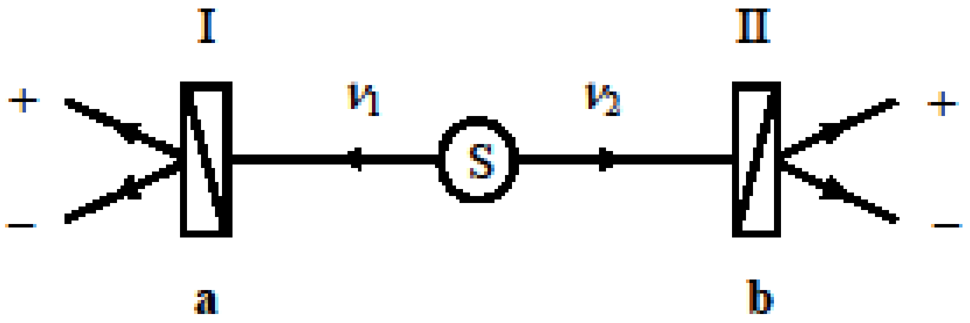
- The source wavefunction is a sum of tensor-product states of the 2 photons.
- Suppress all parameters except polarization.
- Basis vectors can be chosen polarized in the *x or* y direction.

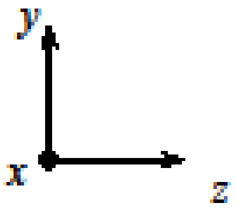
$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|x\rangle\otimes|x\rangle + |y\rangle\otimes|y\rangle\right)$$

Note – this wavefunction **CANNOT** be written as a single tensor product of the form $(\lambda|x\rangle + \eta|y\rangle) \otimes (\lambda' |x\rangle + \eta'|y\rangle)$.

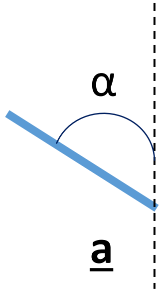
We say that $|\Psi\rangle$ is **entangled**. Entanglement is essential to the violation of Bell′s theorem and to the modern developments of quantum engineering.

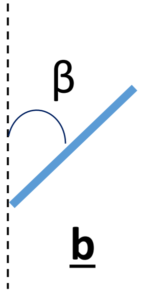
Quantum predictions





$$P_{++}(\alpha,\beta) = P_{--}(\alpha,\beta) = \frac{1}{2}\,cos^2(\alpha-\beta)$$
$$P_{+-}(\alpha,\beta) = P_{-+}(\alpha,\beta) = \frac{1}{2}\,sin^2(\alpha-\beta)$$





Example: When the two polarizers are set *the same*, the probability of **<u>the same</u>** outcome is

$P_{++}(\alpha,\alpha) + P_{--}(\alpha,\alpha) = cos^2(0) = 1$

In other words, "whenever the first polarizer yields +, then so does the second and similarly with –".
Just like the Mermin setup.

Addressing the locality loophole

- Early experiments set the polarizer positions and then took a sequence of measurements
- Objection: there was time for polarizers to theoretically transmit information to one another
- Aspect experiments: “randomly” switch polarizer positions faster than time of photon flight

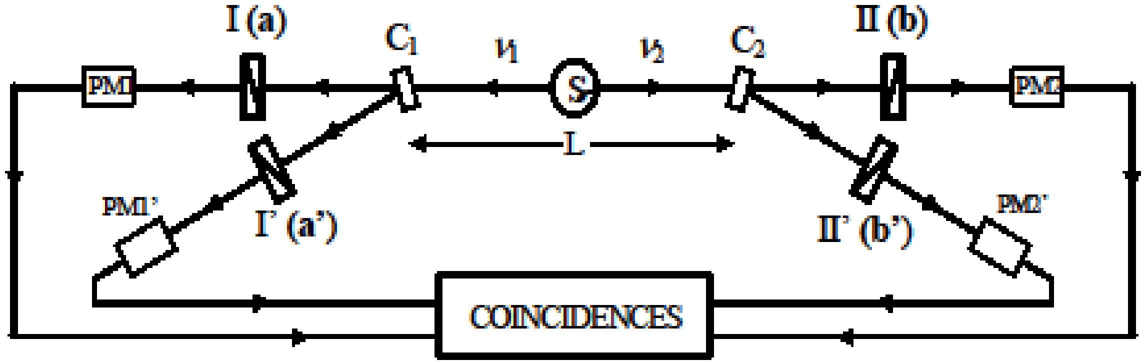


Figure 15 - Timing-experiment with optical switches (C_1 and C_2). The switch C_1 followed by the two polarizers in orientations a *and* a' *is equivalent to a single polarizer switched between the orientations* a *and* a'. *A switching occurs approximatively each* 10 ns. *A similar setup, independently driven, is implemented on the second side.In our experiment, the distance* L *between the switches was large enough* (13 m) *that the time of travel of a signal between the switches at the velocity of light* (43 ns) *was significantly larger than the delay between two switchings (about* 10 ns *) and the delay between the emission between the two photons* (5 ns *average).*

The switching of the light was effected by home built devices, based on the acousto-optical interaction of the light with an ultrasonic standing wave in water. The

Summary and progress

- Einstein argued that QM couldn't be a complete description of reality
- Bohm etc. re-phrased "reality" as a *local hidden variables* theory of nature
- Bell's inequality: any local hidden variables model of an Aspect experiment obeys

$$-2 \leq S(a,a',b,b') \leq 2$$

- The Aspect experiments violate these inequalities
  - So nature can't be described by a local hidden variables theory
- The Aspect experiments agree with QM
- Improvements address potential loopholes
  - Locality
  - Background coincidence counts (accidental synchronicity)
  - Collimation