A GENTLE INTRODUCTION
TO NEUTRINO OSCILLATIONS
Lecture 1: Short review of neutrino physics

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Chapter 1

SHORT REVIEW OF NEUTRINO PHYSICS

1.1 Neutrino discovery

The discovery of radioactivity at the end of the 19th century brought about a number of interesting paradoxes. One of them was the paradox of energy non-conservation in beta radioactivity [1], i.e., in types of nuclear decays that result in emission of electrons.

Let us explain this paradox using the neutron beta decay as an example.[1] Suppose that neutron decays into two particles: a proton and an electron:

$$n \to p^+ + e^-. \tag{1.1}$$

If the initial neutron is at rest, its total momentum is zero and its total energy is $m_n c^2$. The momentum conservation law then requires that proton and electron momenta have equal magnitudes and opposite directions: $\boldsymbol{q}$ and $-\boldsymbol{q}$, respectively. Then relativistic energies of the two decay products are

$$E_p = \sqrt{m_p^2 c^4 + q^2 c^2} \approx m_p c^2, \tag{1.2}$$

$$E_e = \sqrt{m_e^2 c^4 + q^2 c^2} = m_e c^2 + E_{kin}. \tag{1.3}$$

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[1] We use the example of neutron decay, being fully aware that the mentioned controversy was brewing in the 1920's, i.e., before the discovery of the neutron.

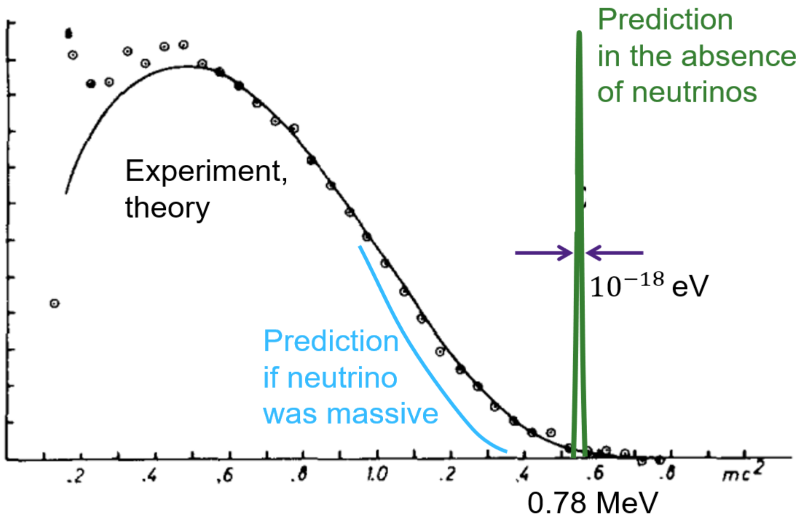

Figure 1.1: Energy spectrum of electrons produced in neutron decay [2]

In (1.2) we took into account that $q \ll m_p c$, therefore proton kinetic energy is much smaller than its rest energy $m_p c^2$. And in (1.4) we explicitly indicated electron kinetic energy E_{kin}, which can be measured experimentally [2]. From the energy conservation law it follows

$$m_n c^2 \approx m_p c^2 + m_e c^2 + E_{kin}, \tag{1.4}$$

and

$$E_{kin} \approx (m_n - m_p - m_c)c^2 = 0.78 MeV. \tag{1.5}$$

This means that electrons emitted in reaction (1.1) must have a very narrow spectrum of energies centered around the value of 0.78 MeV, as shown by the green line in Fig. 1.1. The width of this peak may be estimated from neutron's lifetime $\Delta t = 878s$ by using the time-energy uncertainty formula

$$\Delta E \approx \frac{\hbar}{\Delta t} = 10^{-18} eV. \tag{1.6}$$

In spite of this prediction, experimental spectra of beta-decay electrons were found to be broad continuous distributions extending from zero to the threshold value E_{kin} as show by the solid black line in Fig. 1.1.

Beginning of the 20th century was marked by the advent of relativity and quantum mechanics, which, seemingly, denied all rules of the old classical physics. Niels Bohr was ready to "explain" this paradox by assuming that the energy conservation law is no longer applicable to subnuclear phenomena. However, this point of view was opposed by Wolfgang Pauli who advanced an idea that one more particle[2] is emitted together with the proton and the electron in beta decays. So, according to Pauli, the decay equation should be rewritten as

$$n \to p^+ + e^- + \tilde{\nu}_e. \tag{1.7}$$

The assumption was that the antineutrino carries away a portion of the kinetic energy, but escapes undetected. In this case the kinetic energy of the electron is no longer fixed at the value E_{kin}, but is allowed to vary in the broad interval $(0, E_{kin})$ in agreement with observations. Three immediate conclusions follow from this proposal: i) neutrino must be neutral (hence its name); ii) its interactions with other particles must be very weak;[3] iii) neutrino mass must be very small.

The latter point becomes obvious if we take a closer look at the experimental spectrum in Fig. 1.1. If the antineutrino had a noticeable mass m_ν then instead of extending up to the threshold of $0.78MeV = (m_n - m_p - m_e)c^2$, the electron energy spectrum should have terminated at a lower value of $(m_n - m_p - m_e - m_\nu)c^2$. See blue line in Fig. 1.1.

For a long time neutrino was assumed to be massless. However, as we will see later, the phenomenon of neutrino oscillations indicates that these particles have nonzero masses. Their exact values are still not known. Measurements of the endpoints of beta-electron spectra [3] can tell only that the mass of the electron (anti)neutrino does not exceed 0.8 eV/c^2.

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[2] later dubbed *electron antineutrino* $\tilde{\nu}_e$

[3] Now we know that neutrinos can easily travel through the bulk of the Earth without experiencing a single interaction event.

1.2 Lepton numbers

Further important discoveries were made in 1960's. It was found out that there are three charged massive leptons: electron e^-, muon μ^- and tauon τ^-. Correspondingly, neutrinos also come in three flavors: electron neutrino ν_e, muon neutrino ν_μ and tau neutrino ν_τ. See table 1.1.

Table 1.1: Leptons and their lepton numbers.

Neutrino	Heavy lepton	Electron number	Muon number	Tauon number
ν_e	e^-	1	0	0
ν_μ	μ^-	0	1	0
ν_τ	τ^-	0	0	1

It appeared convenient to introduce so called *lepton numbers* (or charges): electron number, muon number and tauon number, because for a long time it was assumed that they are strictly conserved quantities. For example, electron e^- and electron neutrino ν_e have electron lepton number equal to $L_e = +1$ while positron e^+ and electron antineutrino $\tilde{\nu}_e$ have $L_e = -1$. Then the law of conservation of the electron lepton number requires, for example, that in a neutron decay (1.7) an electron antineutrino should be emitted together with the electron. Only in this case the sum of lepton numbers is equal (to zero) on both sides of equation (1.7).

Another illustration of the lepton number conservation can be found in decays of positive pions

$$\pi^+ \to \mu^+ + \nu_\mu. \tag{1.8}$$

Here the emitted antimuon μ^+ is accompanied by a muon neutrino ν_μ. Then the total muon number on the right hand side is zero, i.e., the same as on the left hand side.

Another example: high energy tau neutrinos may interact with a nucleus (N) and produce negatively charged tau leptons together with other products (X)

$$\nu_\tau + N \to \tau^- + X. \tag{1.9}$$

This reaction illustrates conservation of the tau lepton charge.

1.3 Neutrino oscillations

Conservation of lepton numbers was regarded as a strict law of nature for quite some time. Indeed, one can say with confidence that four major interactions in nature (strong, electromagnetic, weak and gravitational) do respect this conservation law.[4]

First hints that lepton numbers may not be conserved, came from theory. In 1957 Bruno Pontecorvo suggested the idea of neutrino oscillations. In his original model [4], the particle oscillated between neutrino and antineutrino species $\nu \leftrightarrow \bar{\nu}$. This was similar to the neutral kaon oscillation $K^0 \leftrightarrow \bar{K}^0$, which was well established at the time. Later on, his model was elaborated [5] to include the possibility of oscillations between different neutrino flavors.

Neutrino oscillations were regarded as a purely theoretical possibility until 1960's when experimentalists learned how to catch neutrinos coming to Earth from the Sun. The first experiment that detected Solar neutrinos ran from 1970 to 1994 [6]. The experimental facility was setup in an old gold mine in South Dakota. A 380 cubic meter tank was placed 1.5 km below the ground and filled with perchloroethylene, which is a common dry-cleaning fluid rich in chlorine. See Fig. 1.2.

The idea was that electron neutrino coming from the Sun would interact with a chlorine atom to produce argon and electron, according to reaction[5]

$$\nu_e +^{37} Cl \to^{37} Ar^+ + e^-. \tag{1.10}$$

According to theoretical models of Solar nuclear reactions, the flux of electron neutrinos on Earth should be 7×10^{10} particles per square centimetre per second. However, the probability of reaction (1.10) is so low that despite large amount of chlorine in the tank, only few tens of ^{37}Ar atoms were extracted from the liquid every few weeks.[6]

Remarkably, the number of detected argon atoms was about one-third of that predicted by theoretical models of the Solar interior and nuclear reactions going on there. Initially, many doubts existed about the accuracy of these models and the reliability of experimental results. But after many

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[4] Note that all neutrino reactions (1.7) - (1.9) mentioned in the previous section are examples of the weak interaction.

[5] This is another example of electron lepton number conservation: the electron number is equal to one on both sides of this equation.

[6] Such small quantities of ^{37}Ar atoms could be detected thanks to their radioactivity.

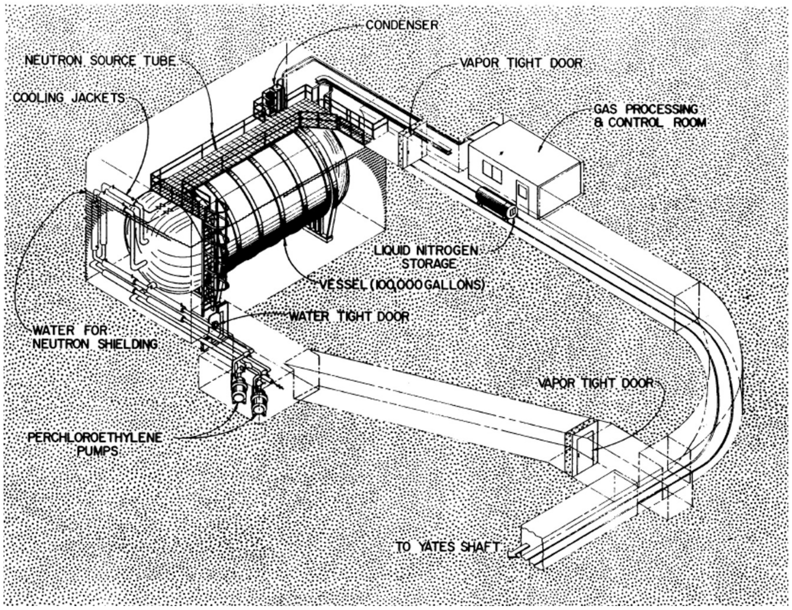

Figure 1.2: Homestake solar neutrino experiment [6].

years of careful analysis, it was concluded that both theory and measurements are essentially correct and the deficit of neutrinos is real. Eventually, the following explanation was agreed upon: Nuclear reactions inside the Sun produce electron neutrinos ν_e. During their travel toward the Earth, these neutrinos oscillate and partially convert to ν_μ and ν_τ flavors, which do not interact with chlorine nuclei and therefore escape detection.

The idea about neutrino oscillations was later confirmed by multiple experiments, which studied Solar neutrinos as well as neutrinos produced in nuclear reactors and accelerators. For example, OPERA experiment [7] used SPS accelerator at CERN in Switzerland to produce a 400 GeV/c proton beam. These protons were directed at a graphite target, as shown by the arrow on the left hand side of Fig. 1.3(a). This resulted in production of positive pions and kaons whose sample trajectory is marked by the red line in the figure. Then pions decayed according to reaction (1.8) with emission of μ^+ (yellow line) and a high energy muon neutrino (broken line).[7]

The beam of neutrinos was directed to LNGS laboratory in Italian Gran Sasso 732 km away from CERN. See Fig. 1.4. This facility had a huge

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[7] Kaons decayed by a similar mechanism $K^+ \to \mu^+ + \nu_\mu$.

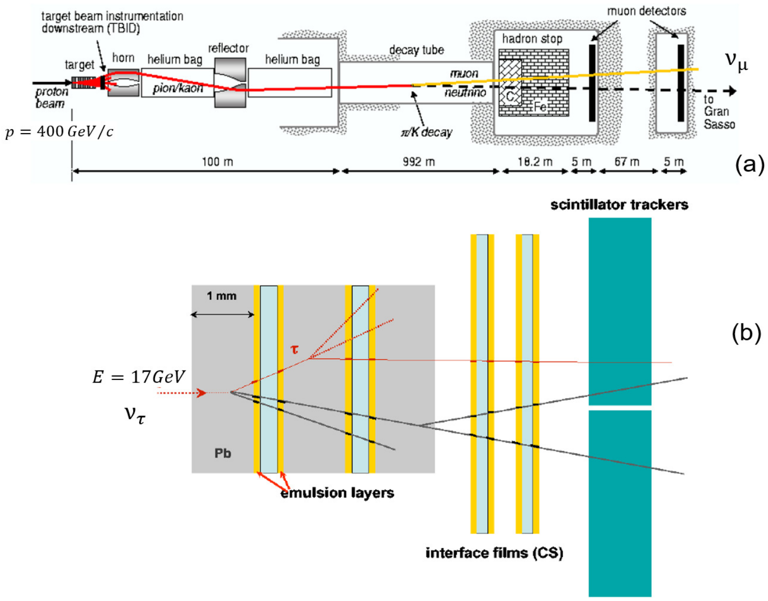

Figure 1.3: OPERA experiment [7]: (a) Production of ν_μ neutrinos at CERN; (b) detection of a $\nu_\tau + N \to \tau^- + X$ event at Gran Sasso site.

and elaborate neutrino detector shown in Fig. 1.5. Basically, the detector consisted of stacked layers of lead and photo-emulsion, as shown in Fig. 1.3(b). It was expected that neutrinos in the beam would interact with lead nuclei and produce secondary particles. The reaction products would leave their marks in emulsion layers downstream. These marks could be traced back to the original interaction vertex, so that identities of the reaction products, their energies, etc. could be determined. Experimentalists were especially focused on finding events described by reaction (1.9) in which τ^- leptons are produced among other particles (see Fig. 1.3(b)), because such events would indicate the presence of τ-neutrinos in the beam and constitute a direct proof of flavor oscillations.

The data were collected between 2008 and 2012. Only five $\nu_\tau \to \tau^-$ events were observed in total. This was roughly consistent with expectations and proved that during their travel from CERN to Gran Sasso some neutrinos converted from ν_μ to ν_τ flavor.

Current understanding of neutrino properties allows us to calculate the oscillation pattern rather accurately. For example, Fig. 1.6 plots probabilities for finding the three neutrino flavors in OPERA-like experiment. It appears

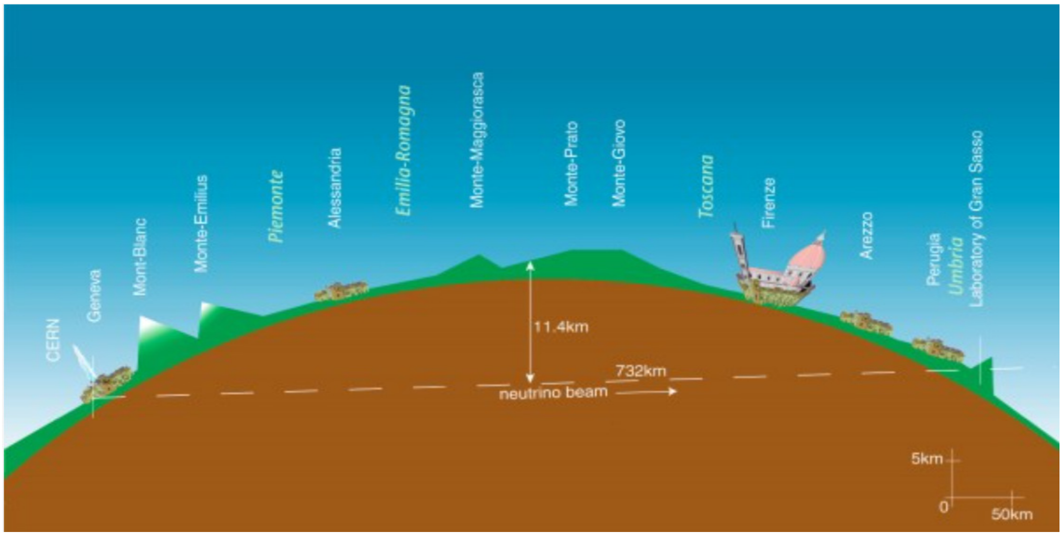

Figure 1.4: OPERA experiment

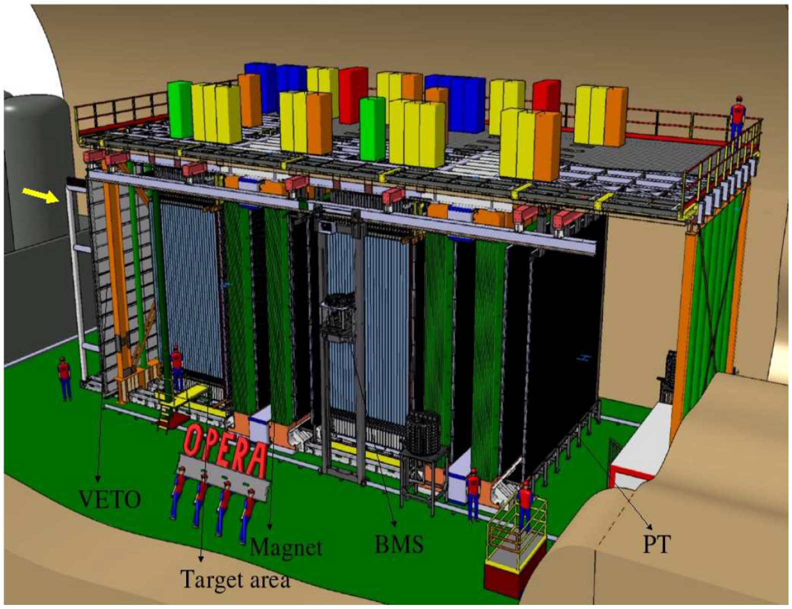

Figure 1.5: Artistic view of the OPERA detector

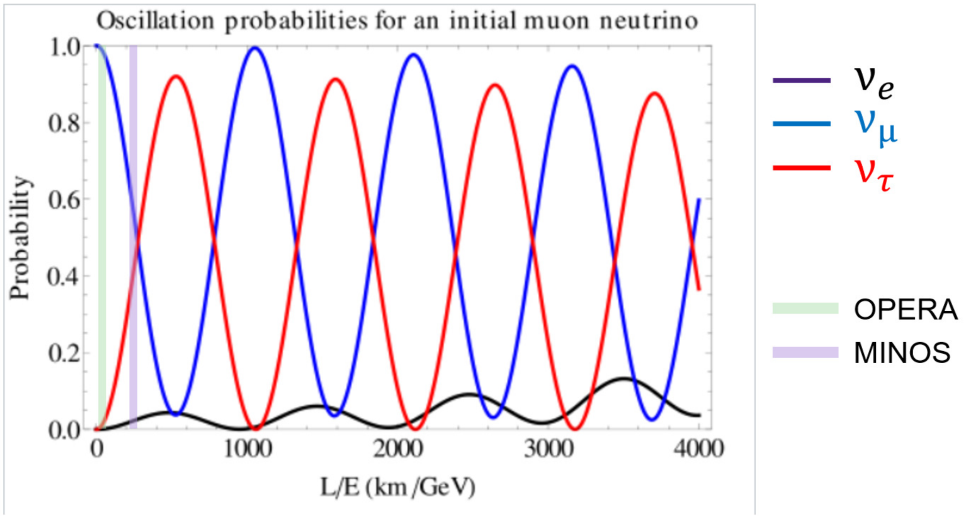

Figure 1.6: Oscillation probabilities for an initial muon neutrino. From Wikipedia article "Neutrino oscillation". L/E intervals for two accelerator experiments are also shown: OPERA [7] and MINOS [8].

that probabilities depend on the parameter L/E, where L is the distance between neutrino source and detector and E is the neutrino energy. As expected, the plot begins from a pure μ-neutrino state at the source ($L/E = 0$):

$$P(\nu_\mu) = 1, \qquad P(\nu_e) = P(\nu_\tau) = 0.$$

As L/E parameter grows, the three probabilities perform a complicated dance, while the total probability remains constant:

$$P(\nu_\mu) + P(\nu_e) + P(\nu_\tau) = 1$$

For low values of L/E, ν_μ and ν_τ flavors are dominant as they oscillate in almost sinusoidal fashion, while probability of ν_e stays low. At higher L/E values, $P(\nu_e)$ increases and the dance becomes more complicated.

As we mentioned in the beginning of this section, all four major forces in nature preserve neutrino flavors. Thus observed oscillations indicate the presence of a new neutrino-mixing force. In the rest of this work, we will explain how this fifth force works and how time-dependent neutrino probabilities should be calculated and interpreted.

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