Derivation of Galilean transformation

Eugene Stefanovich

May 25, 2024

Let $\psi(p)$ be a momentum-space wavefunction. The corresponding position-space wave function is

$$\psi(r,t) = \int \psi(p) e^{ipr} e^{-ip^2 t/(2m)} dp$$

In the moving frame momentum-space function transforms to

$$\Psi(p) = \psi(p - mv)$$

The corresponding position-space function is

$$\begin{aligned}
\Psi(r,t) &= \int \psi(p-mv) e^{ipr} e^{-ip^2 t/(2m)} dp \\
&= \int \psi(q) e^{i(q+mv)r} e^{-i(q+mv)^2 t/(2m)} dq \\
&= \int \psi(q) \left(e^{iqr} e^{imvr} e^{-iq^2 t/(2m)} e^{-imv^2 t/2} e^{-iqvt} \right) dq \\
&= e^{imvr} e^{-imv^2 t/2} \int \psi(q) \left(e^{iq(r-vt)} e^{-iq^2 t/(2m)} \right) dq \\
&= e^{imvr} e^{-imv^2 t/2} \psi(r - vt, t) \qquad (1)
\end{aligned}$$

In notation adopted in [1]

$$\begin{aligned}
\lambda &= mc^2 \\
\gamma &= \frac{c^2}{\lambda} = \frac{1}{m}
\end{aligned}$$

eq. (1) takes the form

$$\Psi(r,t) = \psi(r - vt, t) \exp\left[\frac{i}{\gamma}\left(vr - \frac{v^2 t}{2}\right)\right]$$

which is eq. (1.80) in [1]

References

[1] B. Svistunov, E. Babaev, and N. Prokof'ev. *Superfluid States of Matter*. CRC Press, 2022.