Rotating Black Holes

October 7, 2025

Outline

- A bit of history.
- Talk 1: Qualitative description of rotating (Kerr) black holes
  - Overview of Kerr spacetime geometry
  - Relationship among structural elements
  - Structural dependence on angular momentum
  - Motion in Kerr spacetime
  - Visualizing the complete Kerr spactime manifold
  - Astrophysical black holes.
  - References.
- Next: Quantitative description of rotating black holes

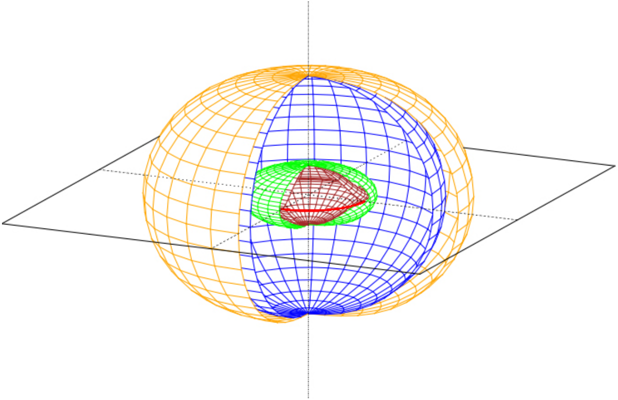
A Bit of History

The theoretical basis for black holes are particular, exact solutions of Einstein's field equations, characterized by only three parameters: mass (m), charge (q) and angular momentum (J), as shown in the following table:

Solution	Date	m	q	J
Schwarzschild	1916	Y	N	N
Reisner-Nordstrom	1916-1921	Y	Y	N
Kerr	1963	Y	N	Y
Kerr-Newman	1965	Y	Y	Y

- Solutions are idealized - the black hole is the only thing in the universe.
- Consequently, the corresponding spacetimes are asymptotically flat - indistguishable from Minkowski spactime far from the black hole.
- Solutions are named after the people who discovered them, as are the corresponding black holes and their entire spacetimes.
- Thus, we speak of Schwarzschild spacetime and Schwarzschild black holes or Kerr spacetime and Kerr black holes, etc.

Overview of Kerr Spacetime Geometry

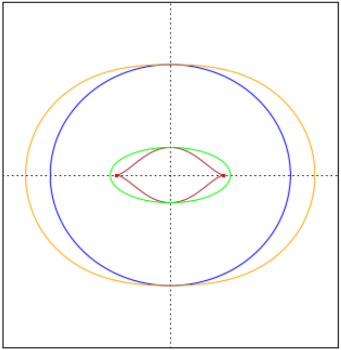


Structural elements:

- Outer stationary limit (s_+)
- Outer horizon (r_+)
- Inner horizon (r_-)
- Inner Stationary Limit (s_-)
- Ring singularity (R)

Relationship Among Structural Elements

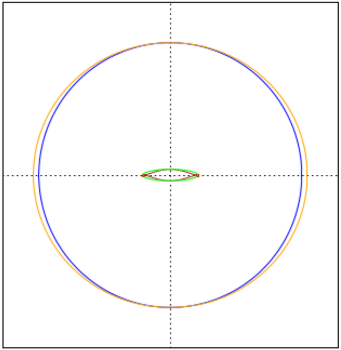
We look at polar cross-sections of the Kerr geometry, because it's easier to discern the relationships among the structural elements.
Note: all polar cross-sections are equivalent, due to axial symmetry.



Relationships:

- Outer stationary limit is outside the outer horizon, except at the poles, where it is tangent.
- Inner stationary limit: is inside the inner horizon, except at the poles, where it is tangent.
- Ring singularity lies in the equatorial plane, where it coincides with the boundary of the inner stationary limit.

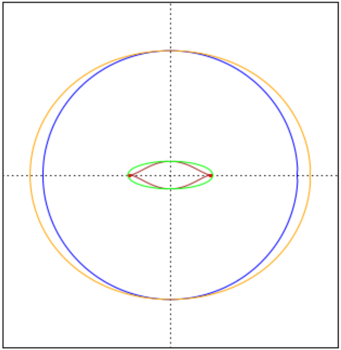
Structural Dependence on Angular Momentum



For moderate angular momentum:

- Outer horizon is nearly circular.
- Outer stationary limit nearly coincides with outer horizon.
- Inner horizon is very small and nearly coincides with inner stationary limit.
- Ring singularity is very small, as well.

Structural Dependence on Angular Momentum



With increasing angular momentum:

- Outer stationary limit separates further from the outer horizon.
- Outer horizon ellipsoid becomes flatter at the poles and wider at the equator.
- Ring singularity, inner stationary limit and Inner horizon grow large, relative to outer horizon.

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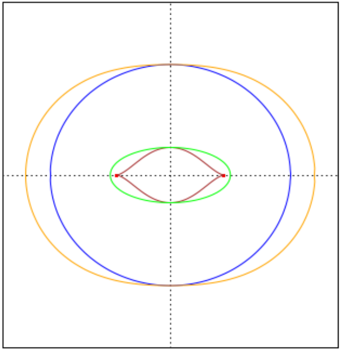

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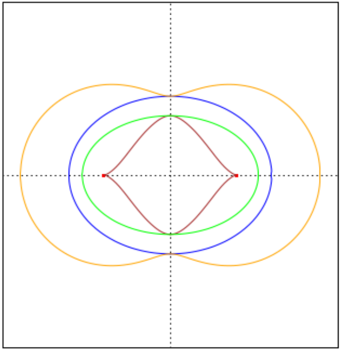
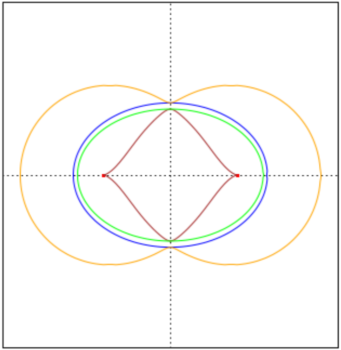


For large angular momenta:

- Outer stationary limit develops lobes.
- Outer horizon and Inner horizon become increasingly alike in size and shape.

Structural Dependence on Angular Momentum



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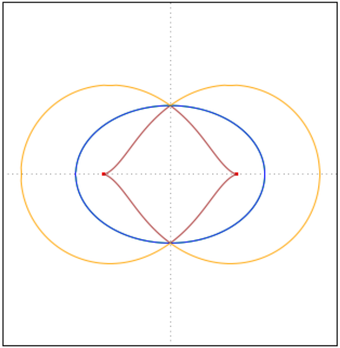

At maximum angular momentum:

- The two horizons coincide.
- This is called an *extremal* black hole.
- Larger angular momenta are assumed to be prohibited, because ...
  - No horizons exist.
  - Singularity is exposed to external observers - a *naked singularity*.
  - Violates *cosmic censorship conjecture* (Roger Penrose, 1969).

Motion in Kerr Spacetime

Preliminaries:

- Motion is affected by:
  - The inward pull of gravity.
  - *Frame dragging* - the GR effect whereby a rotating mass drags the surrounding spacetime, carrying local inertial frames with it, such that freely falling bodies acquire a co-rotational drift, as seen by distant, stationary observers.
- We consider a spacecraft released from rest with its engines off (free fall), at a distant, off-axis location.
  - The spacecraft can turn its engines on at any time.
  - With engines off, spacecraft moves on a time-like geodesic.
  - With engines on, spacecraft can determine its own time-like trajectory (but choices may be limited).
- We will see that an understanding of motion reveals the significance of the structural elements discussed above.

Motion in Kerr Spacetime (continued)

From release point to just above the outer stationary limit:

- Engines off - spacecraft at the mercy of both inward pull and frame dragging.
  - Distant, stationary observers see spacecraft follow inward spiral.
  - Freely falling observers (crew):
    - See their own motion as radially inward.
    - See motion of distant, stationary observers as having retrograde rotation.
    - Can determine their rotation by measuring gyroscopic precession (*Lense-Thirring effect*, 1918).
  - Spacecraft inevitably reaches and crosses outer stationary limit at a point along its time-like geodesic.
  - Angular velocity increases as spacecraft gets closer to the black hole..
- Engines on - spacecraft can compensate for both inward pull and frame dragging. Thus, spacecraft can
  - Move inward or outward.
  - Move with or opposite to black hole rotation.
  - Remain stationary.
  - Reach and cross the outer stationary limit, only if desired.

Motion in Kerr Spacetime (continued)

From the outer stationary limit to just above the outer horizon:

- Engines off - spacecraft continues inexorable inward motion.
  - Distant, stationary observers see spacecraft ...
    - Continue inward spiral.
    - Approach the outer horizon, a but never reach it.
    - Attain angular velocity matching that of the black hole.
  - Crew sees spacecraft
    - Continue inward radial motion, etc.
    - Reach the outer horizon in finite time, at a point along its time-like geodesic.
- Engines on - spacecraft can no longer compensate for frame dragging. Thus, spacecraft
  - Can still move inwards or outwards, including escaping to region above the outer stationary limit.
  - Cannot move opposite to black hole rotation.
  - Cannot even remain stationary (thus "stationary limit").
  - Thus, spacecraft can only spiral inwards or outwards.

Motion in Kerr Spacetime (continued)

From the outer horizon to just above the inner horizon:

- Distant, stationary observers cannot observe this part of the spacecraft's journey.
- Engines off - crew
  - Sees continued inward radial motion, etc.
  - Cannot send signals to the distant observers.
  - Spacecraft inevitably reaches inner horizon along its time-like geodesic.
- Engines on - crew
  - Can no longer move outwards or send signals to distant observers.
  - Spacecraft still has some choice of time-like paths, but all time-like paths lead to inner horizon.
- Inability to send signals indicates that outer horizon is an *event horizon*.

Motion in Kerr Spacetime (continued)

From the inner horizon to just above the inner stationary limit:

- Engines off - spacecraft inevitably reaches inner stationary limit at point along its time-like geodesic.
- Engines on - spacecraft still has some choice of time-like paths, but all time-like paths lead to inner stationary limit.
- Engines on or off - future becomes indeterminate upon reaching the inner horizon $\Longrightarrow$ inner horizon is a *Cauchy horizon*.

Cauchy horizon - infinitely many different futures are consistent with the same prior history. Normally,

- Spacetime can be viewed as a continuous sequence of *space-like hypersurfaces*.
- Each space-like hypersurface contains all of space at a fixed time.
- Each space-like hypersurface is a *Cauchy surface*.
- The conditions prevailing on any Cauchy surface are sufficient to predict (or retrodict) the conditions on all future (or past) Cauchy surfaces.
- This predictability ends at a Cauchy horizon.

Motion in Kerr Spacetime (continued)

From the inner stationary limit to the equatorial disk that has the ring singularity as its boundary:

- If the spacecraft's trajectory lies in the equatorial plane, it terminates at the ring singularity, i.e. the spacecraft is destroyed.
- Otherwise, the singularity is avoided, i.e. the spacecraft passes through the interior of the disk.
- If engines are on after passing the inner stationary limit, the spacecraft can, once again, compensate for inward pull and frame dragging. Thus, the spacecraft can
  - Remain stationary.
  - Wander about in the region between the inner stationary limit and the disk (a pointless exercise).
  - Pass through the interior of the disk.

What does it mean to pass through the interior of the disk? Where does the spacecraft go?

Motion in Kerr Spacetime (continued)

Beyond the ring singularity:

- Once the spacecraft passes through the interior of the disk,
  - It emerges into one of infinitely many asymptotically flat regions.
  - Which of these, it enters is completely unpredictable.
  - This is a direct manifestation prior passage through the Cauchy horizon.
  - With engines off, geodesic motion carries the spacecraft to spatial infinity.
  - With engines on, the spacecraft can travel on any time-like geodesic, i.e. maneuver freely throughout the region.
- In each of these regions,
  - The ring singularity is NOT hidden behind an event horizon, i.e. it is a naked singularity.
  - Thus, Kerr spacetime violates the cosmic censorship conjecture.

Visualizing the Complete Kerr Spactime Manifold

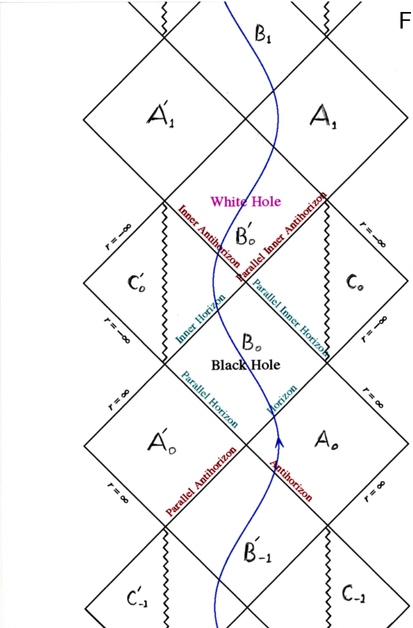

Figure is Penrose diagram which:

- Compresses spatially infinite regions into a finite picture, where
  - Two spatial dimensions are suppressed.
  - Time increases upwards.
  - Spatial coordinate increases rightwards.
  - Null lines (surfaces) are at $\pm 45^o$.
- Depicts an endless lattice of spacetime regions, where
  - A_n, $n \in \mathbb{Z}$ correspond to regions outside the event horizon of identical Kerr black holes.
  - B_n, $n \in \mathbb{Z}$ correspond to regions between the event horizon and Cauchy horizon of those black holes.
  - C_n, $n \in \mathbb{Z}$ correspond to asymptotically flat regions beyond the ring singularity (wiggly lines).
  - Primed letters indicate corresponding white hole regions.
  - Curve winding its way through the lattice is a time-like geodesic.

Astrophysical Black Holes

Recall that the Kerr solution is based on the assumption that the black hole mass is the only thing in the universe. Clearly this cannot be a precise model for real black holes. In reality,

- The energy of in-falling matter increases exponentially as it approaches the Cauchy horizon.
- This is called *mass inflation*.
- As a result, the Cauchy horizon quickly becomes unstable, transforming into a singular surface.
- The geometry beyond is also unstable and ill-defined.
- With the demise of the Cauchy horizon,
  - Determinism is restored.
  - The cosmic censorship conjecture is saved.
- However, recent work (2018) has shown that in a de Sitter universe:
  - Mitigating factors can prevent severe mass inflation.
  - Thus, the Cauchy horizon can survive.
- Our universe is evolving into a de Sitter universe.

References

- The Mathematical Theory of Black Holes, S. Chandrasekhar, 1983.
  - A comprehensive and detailed account of all the types of black holes.
  - A working knowledge of general relativity is expected.
  - First chapter reviews the necessary math.
- The Kerr Spacetime, D. Wilthsire, M. Visser and S. Scott, editors, 2009.
  - A comprehensive overview of Kerr spacetime, with contributions from multiple authors, including Roy Kerr, himself.
- General Relativity, R. Wald, 1984
  - A general-purpose text book that teaches relativity.
  - Covers Schwarzschild and Kerr-Newman black holes, but not nearly has comprehensively as Chandrasekhar.
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