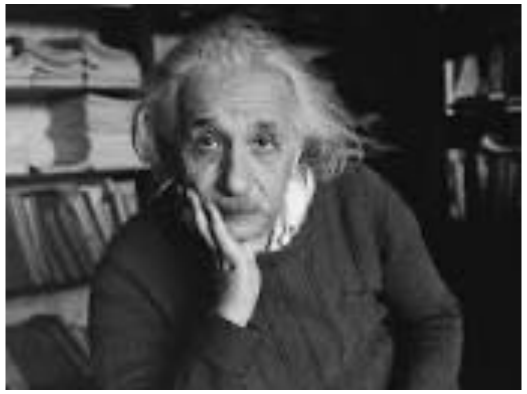
Einstein - special relativity part 1, kinematics

Talk #3 On the Electrodynamics of Moving Bodies



Ques: Why am I nuts enough to give a talk on special relativity when most (all?) of you are far far more deeply into the math of general relativity than I have am?

Ans: I like to believe two things are amazing about Einstein. First is his “technical” brilliance. The second is my interpretation of an aspect of his personality. Call it his arrogance or hubris. I prefer to think of it as simply intellectual courage.

He proposed that time and space are not what everyone had “always known them to be”. And he had the nerve to publish it.

It is well known that that Maxwell’s electrodynamics when applied to moving bodies, leads to asymmetries that do not seem to attach to phenomena. Recall the electrodynamic interaction between a magnet and a conductor. The observable phenomenon depends here only on the relative motion of conductor and magnet. The two cases are either: one moves or the other moves. According to the customary conception the two cases are to be strictly differentiated from each other.

If the maget is in motion and the conductor at rest, there arises a *(moving)* electric field around the magnet which has energy and produces a current in the conductor. If the the magnet is at rest and the conductor is moving, no electric field is created around the magnet, but is created in the conductor and gives rise to a current or charge displacement.

While the supposed fields are not the same, the measurable currents or charge displacements are.

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This is what bothered Einstein: two ways of looking at a situation that gave the same result. Shouldn't there be one?

Examples such as this and the failure to detect a motion of the earth relative to the "light medium" lead to the conjecture that phenomena do not have the concept of "absolute rest". And therefore we can conjecture that all mechanical and electrodynamic laws are equally valid whether or not the system is in motion. We shall raise this conjecture, "the principle or relativity", to the status of a postulate; and shall introduce an additional postulate that the speed of light, C, is independent of the state of motion of the emitting body. The second postulate is only seemingly incompatible with the first.

These two postulates suffice for arriving at a simple consistent electrodynamics of moving bodies on the basis of Maxwell's theory for bodies at rest; with these two postulates the introduction of a "light ether" will prove superfluous.

So what does it mean?

"If a system of coordinates K is chosen so that, in relation to it, physical laws hold in their simplest forms, the same laws hold good in relation to any other system of coordinate K' moving in uniform velocity relative to it."

Albert Einstein

Let's start with the Michelson Morley Experiment

What drove everyone crazy

By "everyone" I mean a couple of physicists

If light is a wave, it is a wave in “something”; no?

It would be very surprising if it did not matter how fast you were going.

The earth moves. Shouldn’t we move at different rates through the “something”?

Think of a swimmer in a river. Shouldn’t you see the swimmer’s speeds to be different if she is swimming upstream, downstream or across?

Or it’s not a river, it’s a lake!!!

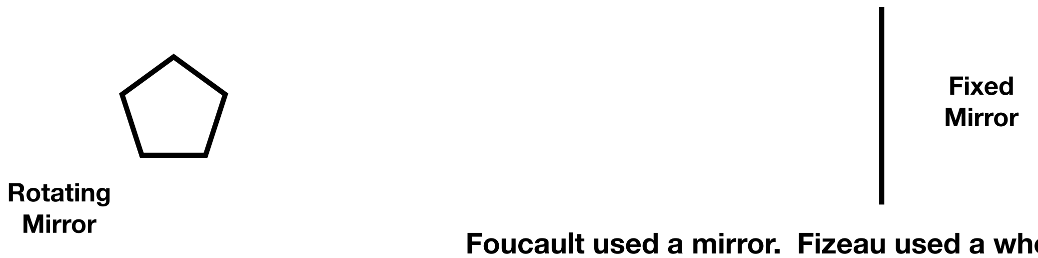
There was a name for the “something”
Luminiferous Aether

If you measure the speed of sound of air and the air is moving, ie there is a wind, the speed you measure is different depending on whether you are measuring in the upwind of downwind direction

Maxwell's equations said light was a wave.

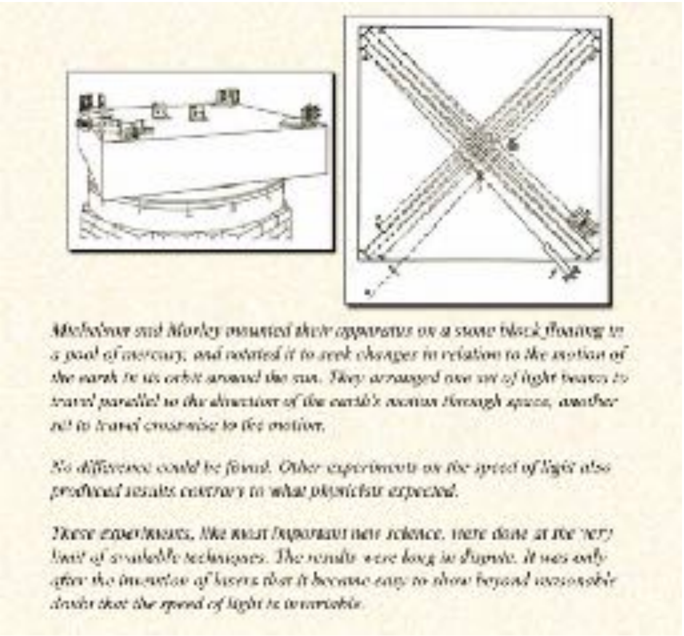
How do you measure the speed of light?

A rotating mirror and a mirror on a mountain and a pulse of light.



Foucault used a mirror. Fizeau used a wheel.

The Michelson-Morley experiment tried to measure small differences, not the precise value.



Michelson and Morley mounted their apparatus on a stone block floating in a pool of mercury, and rotated it to seek changes in relation to the motion of the earth in its orbit around the sun. They arranged one set of light beams to travel parallel to the direction of the earth's motion through space, another set to travel crosswise to the motion.

No difference could be found. Other experiments on the speed of light also produced results contrary to what physicists expected.

These experiments, like most important new science, were done at the very limit of available techniques. The results were long in dispute. It was only after the invention of lasers that it became easy to show beyond reasonable doubt that the speed of light is invariable.

The astounding result was that the speed of light is the same no matter the direction, ie the time of day or year.

"The results were long in dispute." Michelson originally calculated that the shifts should be 0.04 fringes. He measured 0.02 fringes or less. But then someone showed his calculations were not correct and the number should have been about 0.02. Eventually the "or less" proved to be the case.

There were three ways out of the problem:

#1 The earth is the center of everything

#2 Luminiferous Aether “sticks” to the earth

#3 The very concept of space and time needed an update

Lorentz Equations

Physicists first guessed that “time” was somehow changing
Lorentz added that “space” was also changing

The Michelson-Morley experiment was in 1887.

Lorentz and others worked on the theory starting in 1892. He published the final theory in 1904. But he too thought in terms of movement's effect of the ether, ie "stiffening" it.

Einstein thought about it in a completely different way. He gave a new interpretation to time and space. The immediate results were the same, ie the Lorentz equations; just as the currents were the same whether the magnet or the wire moved.

But the concept was revolutionary. Space and time are not what they were thought to be.

<u>And the implication/prediction was spectacular.</u>

The Lorentz Transformation

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\beta = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$\alpha = v/c^2$

Let me remind you what this means with β=1 and α=0.

An event, ☆ ,occurs in a coordinate system a rest. It continues to be at position x and continues for all time t.

Now what does an observer who is moving with velocity *v* to the right see? He sees the event/star move off to the left.

The person standing still sees the coordinates x and t. The person moving sees x'=x-*v*t and t'=t

Classical	Lorentzian
$X' = x - vt$	$x' = \dfrac{x - vt}{\sqrt{1 - \dfrac{v^2}{c^2}}}$
$y' = y$	$y' = y$
$z' = z$	$z' = z$
$t' = t$	$t' = \dfrac{t - \dfrac{vx}{c^2}}{\sqrt{1 - \dfrac{v^2}{c^2}}}$

We are used to this. But let's look at some consequences that might have led Einstein to think about this and his "principle of relativity" that physical laws <u>all</u> hold in moving systems.

The twin's trip to a star

One person goes out and comes back. Are they the same age after the trip?

Lorentz

The traveler is younger!!

Do it correctly: Think of two trips. And three clocks. His stays on the earth. Another goes out. The third starts at the star and comes back. We know what his clock does. We know her rocket speed and how far it is to the star. We calculate what her clock adds when she reaches the star with the (+v) Lorentz transform. We then calculate how much her (the third) clock adds on the way back with (-v) Lorentz transform.

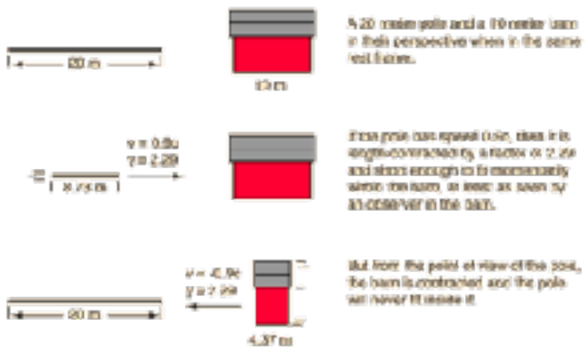
Only she is younger!!

See - Time travel IS possible!!
She travelled into his future

That is the only type possible.
ScFi writers don't like it
Too bad ScFi writers!

Second the barn and pole

Four events:
front of bar, rear of barn
front of bar, front of barn
rear of bar, front of barn
rear of bar, rear of barn



The person with the pole "sees" it cannot fit inside.
The person with the barn "sees" the pole fits.
The order of the events also changes.
"Length contraction" and "time dilation"

Are the “twins” a paradox? Doesn’t she see the earth go “out and back”; so he looks younger to her?

Note: I said three clocks. One of her two goes out and the other comes back. They are synced at the turn. I didn’t say anything about what would happen to her in the acceleration of “turn around”.

Acceleration affects time. Right, Steve?

It took Einstein another twelve years to figure out acceleration!!

The first part - Kinematics

The second part - Elecrtodynamics

Einstein’s genius is seeing that Lorentz’s electrodynamics applied to moving systems has a much deeper meaning.

The first part - Kinematics

#1 Defintion of simultaneity

#2 On the relativity of lengths and time

#3 Theory of transformation of coordinates and time from a system at rest to a system in uniform translational motion relative to it

#4 The physical meaning of the equations obtained concerning moving rigid bodies and moving clocks

#5 Additon theorem of velocities

A quick/superficial review of these points

Two systems K and k with clocks and yardsticks.
k moves to the left with speed v.

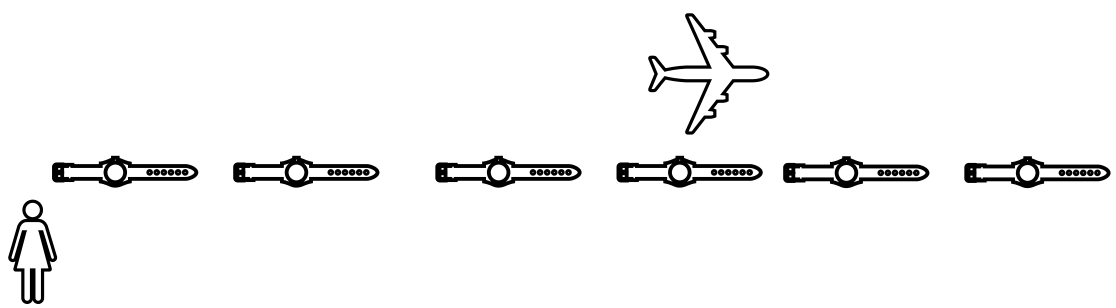
In stationary K with x,y,z,t In moving k with ξ,y,z,τ
These are NOT distances and times. They are readings on yardsticks and clocks of where and when events occur.

Einsteins genius is:

1) the readings ξ,τ are linearly related to x,t.

2) the τ's of three events - a light flash, a reflection and return - can be expressed in terms of the x's and t's of the stationary observer.

3) the relation among the three τ's and the constancy of the speed of light determine the linear coeffients, ie the Lorentz



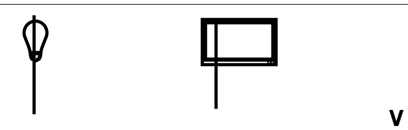
There are two “frames of references”. She is at rest. He is moving. Events occur and they occur in the moving system. One will be a flash of light, say at the tail. The next will be reflection from a mirror at the nose. And the third will be the return of the flash to the tail.

She, very cleverly, has spread synchronized sets of clocks and yard sticks all along the ground under the flight path. So; she can record when and where each happens. He, on the other hand, has is own sets spread out on the floor of the plane.

#1 simultaneity discusses setting two clocks in a system. One makes two readings. Clock A’s first is the time of a flash and its second is the return of the flash from a mirrror. The readings are t_A and t'_A. The other clock, B, makes a reading at the mirror when it sees the flash in the mirror. It’s readin is t_B. (“Reading” means what the clock face says, ie not an elapsed period.) The clocks are synchronous if $t_B - t_A = t'_A - t_B$

Note: $C=R_{AB}/2(t'_A-t_A)$
The universal constant

Readings t_A and then t'_A t_B



Readings t_A and then t'_A t_B

#1 simultaneity/relativity of space and time

Now imagine that the system is in motion to the left. But the clock is synchronous with another clock on the ground at rest. Then

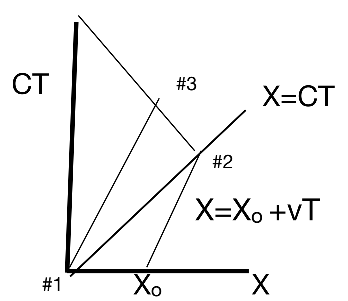
$t_B - t_A = R_{AB}/(C-v)$ and $t'_A - t_B = R_{AB}/(C+v)$

So the observers moving with the clocks do not find the clocks to be synchronous, while the observer on the ground would find all three clocks to be synchronized.

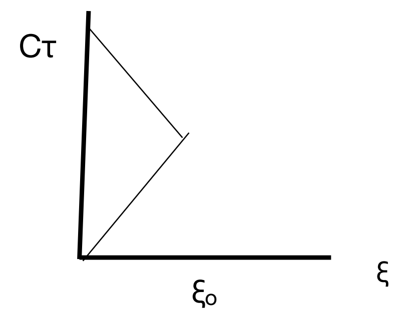
Simultaneous has no absolute meaning for observers in relative motion

#3 Developing the transform. There are two coordinate systems - one at rest and the other moving in the x direction with speed v. Both coordinate systems are three dimensional, of course. The K system is stationary and has clocks and measuring sticks that record values (x,y,z,t) The k moving system has clocks and sticks that record values (ξ,y,z,τ). Let's look at <u>all</u> values of x and t such that X_0=x-vt. This is single point (ie stationary) in the moving moving system.

Einstein makes an assumption - measurements (ie times and positions) in the (ξ,y,z,τ) system are linearly related to the measurements in the (x,y,z,t) system. He also adopts the postulate that the speed of light is the same in both, independent of the velocity of the source. Finally he examines the following: At time τ_0 a flash at the origin. At τ_1 the flash arrives at X_0 and is reflected. Finally at τ_2 the flash arrives back at ξ=0



On the ground
Event#1 T_0=t X_1=0
Event#2 T_1=t+X_o/(C-v) X_2= CX_o/(C-v)
Event#3 T_2=t+X_o/(C-v) + X_o/(C+v)



Moving with X_o
Event#1 τ_0=0 ξ_o=0
Event#2 τ_1=ξ_o/C ξ_2=ξ_o
Event#3 τ_2=2ξ_o/C ξ_3=0

X_o = **X-vt and therefore is a stationary in τ,ξ**

$$\frac{1}{2}\left[\tau(0,0,0,t)+\tau\left(0,0,0,\left\{t+\frac{x'}{V-v}+\frac{x'}{V+v}\right\}\right)\right]=\tau\left(x',0,0,t+\frac{x'}{V-v}\right).$$

The first thing to do is establish the linear relation $\tau = at+bx$ (note: τ depends of x)

Next: Equate: $1/2(\tau_0 + \tau_2) = \tau_1$ and express this in terms of t and x, ie

. Which allows a relation between a and b. The result is:

$\tau = a\{t - vX_o/(C^2-v^2)\}$ or since $X_o = x - vt$ $\tau = a\{C^2/(C^2-v^2)\}[t - vx/C^2]$

This is in the form $\tau = \beta[t - vx/C^2]$, but we have the unknown a instead of β.

Enstein's next step is to look at the perpendicular direction, say y.

$$\frac{1}{2}\left[\tau(0,0,0,t) + \tau\left(0,0,0,\left\{t + \frac{x'}{V-v} + \frac{x'}{V+v}\right\}\right)\right] = \tau\left(x',0,0,t + \frac{x'}{V-v}\right).$$

$1/2\{at + at + a[X_0/(C-v) - X_o/(C+v)]\} = bX_o + at + aX_o/(C-v)$

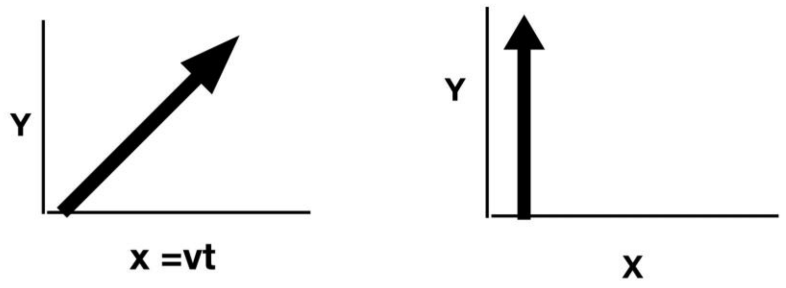
$$1/2\{at+at+a[X0/(C-v)-Xo/(C+v)]\} = bXo + at+aXo/(C-v)$$

$$1/2\{a[X0/(C-v)-Xo/(C+v)]\} = bXo +aXo/(C-v)$$

$$-1/2\{a[X0/(C-v)+Xo/(C+v)]\} = bXo$$

$$-v/(C2-v2) = b/a$$

Next look in the Y direction



Diagonal Velocity is C. X velocity is v. Therefore Y velocity is $(C^2-v^2)^{1/2}$ and $y= t(C^2-v^2)^{1/2}$

The distance is Cτ , that is $y=C\tau$ and therefore $C\tau = t(C^2-v^2)^{1/2}$

From the previous x coordinates you have an expression for τ that has the unknown a. Using that in the above y coordinate expression determines the value of a. The result is

$$\tau = \beta[t-(vx/C^2)] \quad \text{where} \quad \beta = 1/[1-(v/C)^2]^{1/2}$$

The Lorentz Transform

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$y' = y$$

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$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\frac{1}{2}\left[\tau(0,0,0,t) + \tau\left(0,0,0,\left\{t + \frac{x'}{V - v} + \frac{x'}{V + v}\right\}\right)\right] = \tau\left(x',0,0,t + \frac{x'}{V - v}\right).$$

Einstein changed the way we think about time and space. He solved the "problem" of "It looks different if the magnet moves or the wire moves."

They are coordinates of points in four space. The above is from his paper showing the relation of the three points - the flash, the reflection, the return.

Four dimensional notation from his paper - Half the difference between the time of the flash and the return equals the time to the reflection in the moving frame.

#4 The meaning for moving rigid bodies and clocks

A sphere will have dimensions R^2,R^2,R^2 to an observer moving with it. But to a stationary observes its dimensions are $(\beta R)^2,R^2,R^2$. This is dimensional contraction.

The time of a clock in motion is $T = \beta[t-(vx/C^2)]$.
And $x = vt$. So $T = (1/\beta)t$ This time dilation.

Einstein goes on to say that if you move a clock around, it will return with less time elapsed than a clock that didn't move. It took him another 12 years to think through acceleration and gravity.

#5 The addition theorem of velocities.

Start with two systems.
The one on the ground, ie stationary, has coordinates T,X. The other moves to the left velocity v and has coordinates t,x in that system.
Therefore $T= \beta(t+vx/C^2)$ and $X= \beta(x+vt)$

Now imagine something is moving in the moving system and look at it at two points t_1,x_1 and t_2,x_2 .

The next step is to look at the net velocity on the ground. The velocity on the ground is $(X_1-X_2)/(T_1-T_2) = U$. The velocity in the moving system is $(x_1-x_2)/(t_1-t_2)$. Call it w. When you put the expression for U in terms of t and x and do the algebra, the result is:

$$U = (v+w)/(1+vw/C^2)$$

You cannot exceed the speed of light

THIS REPEATS THE LAST SLIDE

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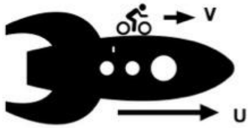


#5 The addition theorem of velocities.

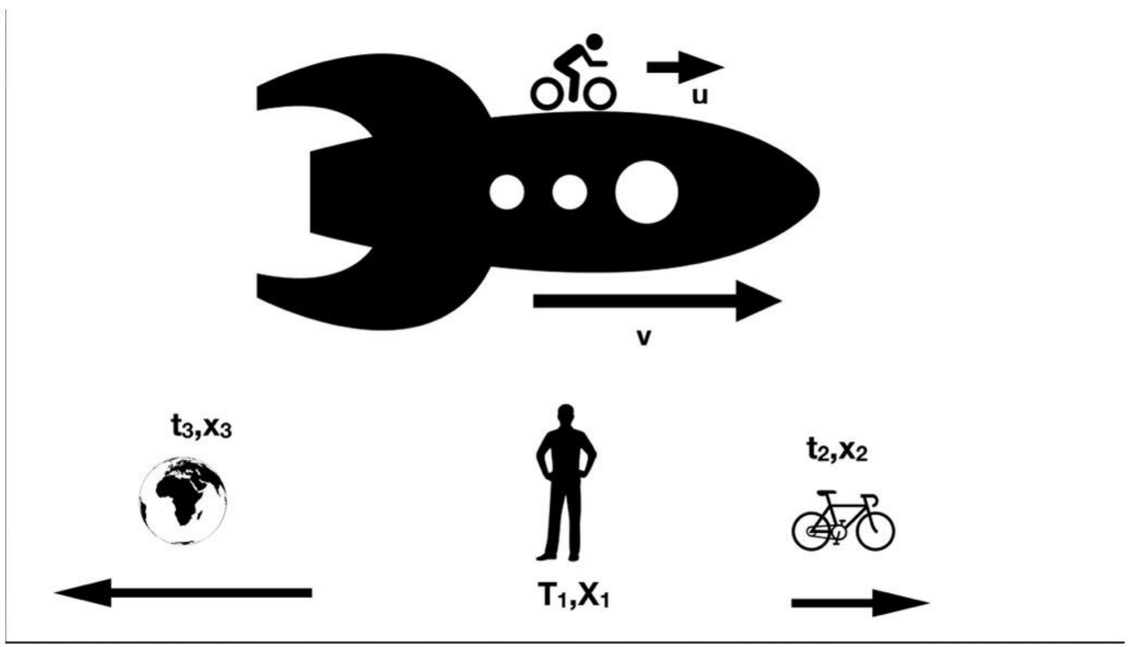
The bike, inside the the rocket, has velocity v. Say v equals 0.7C

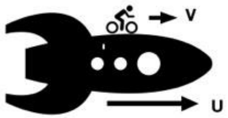
And the rocket ship has velocity u. Say u equals 0.8C.

Qes: Doesn't anyone on the ground see the bike going at W = u+v or 1.5C? Ans: No

$$W = (u+v) / [1+(uv)]$$

$$W = 1.5/1.56 = 0.962$$





#5 The addition theorem of velocities.

The easiest way to derive the addition of vlocities is <u>not</u> to transform from the ground to the rocket and then from the rocket to the bike.
{Einstein doesn't say exactly how he did it.}

The best way to do it is to express T_1,X_1 interms of t_2,x_2 and also express T_1,X_1 interms of t_3,x_3.
Then equate the two expressions.
Then solve for t_3 and x_3 in terms of t_2 and x_2.

What pops out is the expression for the velocity transform. $W = (u+v)/[1+(uv)]$