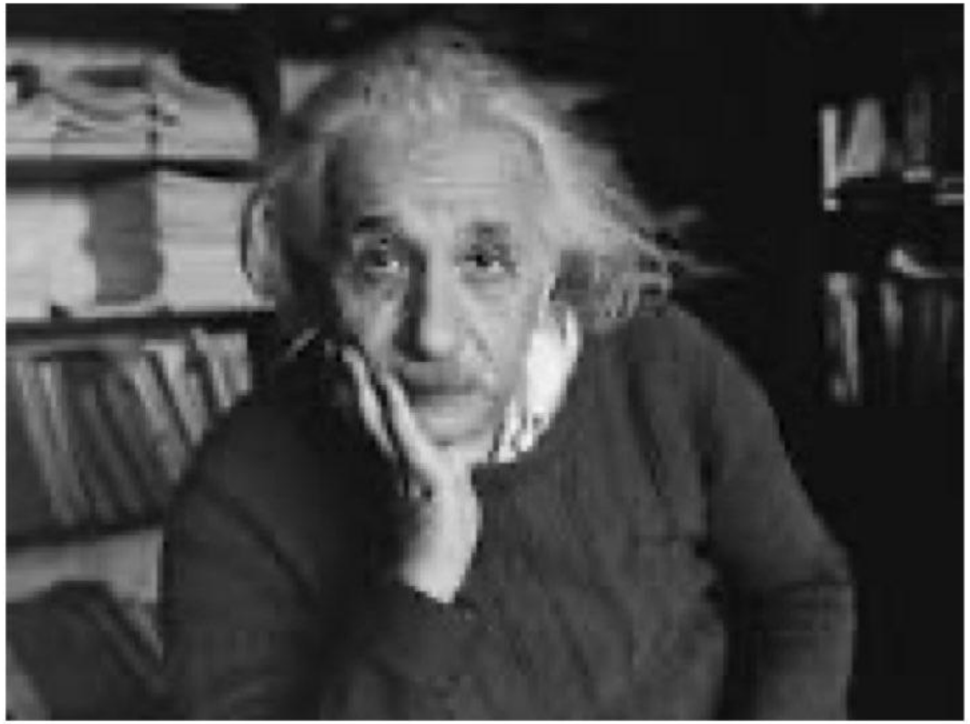
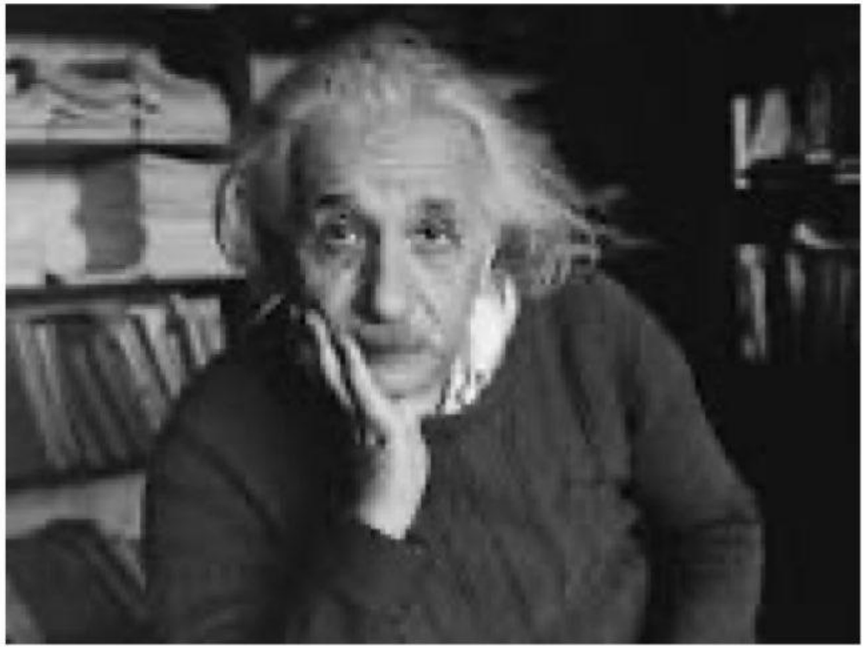
Second series — Talk #2

The transition from a continuous to a quantized world



On a Heuristic point of view concerning the production and transmission of light



Is light; like matter?

Does it too come is lumps?

There exists a profound formal difference between the theoretical conception physicists have formed about gasses and other ponderable bodies and Maxwell's theory of electromagnetic processes in so-called empty space. We conceive of the state of a body as being completely determined by the positions and velocities of the very large but nevertheless finite number of atoms and electrons. We use continuous spatial functions to determine the electromagnetic state of a space, so that a finite number of quantities cannot be considered as sufficient for the complete description of the electromagnetic state of a space.

According to Maxwell's theory, energy is to be considered as a continuous spatial function for all purely electromagnetic phenomena, hence for light. While according to the current conceptions of physicists the energy of a ponderable body is to be described as a sum extending over the atoms and electrons. The energy of a ponderable body cannot be broken up into arbitrarily many, arbitrily small, parts. While according to Maxwell's theory (or more generally according to any wave theory) the energy of a light ray emitted from a point source spreads over a steadily increasing volume.

For matter the energy is in finite chunks, ie the motion of the atoms. For E/M the fields get thinner and thinner.

The wave theory of light, which operates with continuous spatial functions, has proved itself splendidly in describing purely optical phenomena and will probably never be replaced by another theory. But one should keep in mind, however, that optical observations apply to time averages and <u>not</u> to momentary values. It is conceivable that despite the complete confirmation by expriment of the theories of diffraction, refraction, dispersion, etc.; the theory of light, which operates with continuous spatial functions, may lead to contradictions when applied to the phenomena of production and transformation of light.

Indeed, it seems to me that the observations regarding “black-body radiation”, photoluminescence, production of cathode rays by ultraviolet light and other groups of phenomena with the production or conversion of light can be better understood if one asssumes that the energy of light is discontinuously distributed is space.

According to the assumption to be contemplated here, when a light ray is spreading from a point, the energy is not distributed continuously over ever-increasing spaces, but consists of finite number of energy quanta that are localized in points in space, move without dividing and can be absorbed or generated only as a whole.

Part #1 On a difficulty encountered in the theory of "black-body radiation"

Two classical theories play a role in this talk. James Clerk Maxwell made key contributions to both. The first is the description of dynamic electric and magnetic fields. The second is statistical mechanics (S/M). Most physicists accepted S/M, but some did not.

Einstein's contributions as described in both the first and in this talk was to connect S/M to directly observable phenomena.

Maxwell's equations for E/M waves

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

The energy density is $U = (E^2 X B^2)/(8\pi)$

The Poynting vector is $S = C(E X B)/(4\pi)$

The units are from Jackson

The energy density is $U = (E^2 X B^2)/(8\pi)$

The Poynting vector is $S = C(E X B)/(4\pi)$

The Key Point:

Energy does <u>not</u> depend on frequency;

<u>only</u> on the field amplitude!!!

The other theory is statistical mechanics.

The examples are particles. I use them because I can copy the math from published literature since I don't have the programs to do on my computer.

The analogy is that for the energy goes as the square of the field strenght for a single E/M wave and the square of velocity for single particle.

Particles have speed and direction.
E/M waves have amplitude and direction; Also frequency
E/M energy density is per unit volume or per unit frequency.

Statistical mechanics is is full of tricks and surprises. First you devide up the energy in little amounts. A thing can have 1 or 2 or 3 or 4 etc. etc. amounts of energy.

Then you find the most random, ie numerous, way of distributing amounts of energy among a bunch of things. You actually maximize the log of the number of ways.

Before you find most random distribution, you include the constraint that the total amount of energy is fixed. You use LaGrange multipliers, ie strange multipliers of zero.

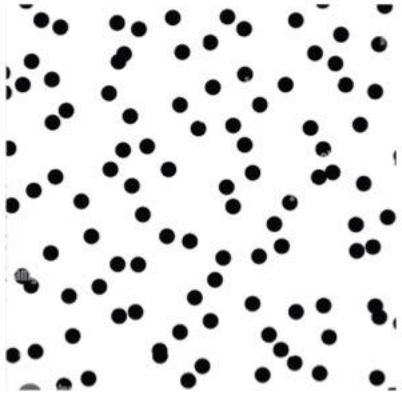
The final step is to let the small energy increments become continuous. The surprise is that the Lagrange multiplier turns out to contain much of the physics.

The results are:

The frequency (or relative probability) of the energy of a mode is exp-(E/kT)

The mean value of a mode is 1/2(kT)

$$e^{-(E/kt)}$$



The atoms of a gas can have energy 1e or 2e or 3e. The number that have 4e is N_{4e} That is, a particle is in box 4e, along with N_{4e} -1 others.

In general, the number of distinct arrangements of N particles into n groups containing $N_1, N_2, \ldots, N_i, \ldots, N_n$ objects becomes:

$$\frac{N!}{N_1! \cdot N_2! \cdot \ldots \cdot N_i! \cdot \ldots \cdot N_n!}$$

Where N_i is the number of objects in container i

The Multiplicity Function:

$$Q(N_1, N_2, \ldots, N_i, \ldots, N_n) = \frac{N!}{N_1! \cdot N_2! \cdot \ldots \cdot N_i! \cdot \ldots \cdot N_n!} = \frac{N!}{\prod_{i=1}^{n} N_i!}$$

There are two physical constraints on our ***classical*** system:

1. the total number of particles must be conserved
2. the total energy of the system must be conserved

Constraint 1 implies:

$$\phi = \sum_{i=1}^{n} N_i = N$$

← Constant

Constraint 2 implies:

$$\psi = \sum_{i=1}^{n} E_i N_i = U$$

← Constant

where U is the total energy for the system

$$\ln Q = N \ln N - N + \sum_{i=1}^{n} N_i \ln g_i - \sum_{i=1}^{n} N_i \ln N_i + \sum_{i=1}^{n} N_i$$

$$\frac{\partial}{\partial N_j} \ln Q + \alpha \frac{\partial \phi}{\partial N_j} - \beta \frac{\partial \psi}{\partial N_j} = 0$$

Substituting in *ln Q* and Constraints 1 and 2:

$$\frac{\partial}{\partial N_j}\left(N \ln N - N + \sum_{i=1}^{n} N_i \ln g_i - \sum_{i=1}^{n} N_i \ln N_i + \sum_{i=1}^{n} N_i \right) + \alpha \frac{\partial}{\partial N_j}\left(\sum_{i=1}^{n} N_i \right) - \beta \frac{\partial}{\partial N_j}\left(\sum_{i=1}^{n} E_i N_i \right) = 0$$

Taking the derivative, noting that *N* is constant and the only terms that are nonzero are when *i* = *j*:

$$-\left(1 \cdot \ln N_j + N_j \cdot \frac{1}{N_j} \right) + 1 + \alpha - \beta E_j = 0$$

$$-\ln N_j + \alpha - \beta E_j = 0$$

f(E) is the distribution of energy states and
f(E) = (1/kT)exp(-E/kT)

The average energy per mode (ie degree of freedom)

$$E_{avg} = \frac{\int E\, e^{(-E/kT)}}{\int e^{(-E/kT)}}$$

1/2 kT for each mode
3/2 kT for a free particle
kT for an occilator

Why does β have the value 1/(kT)? Or expressed in it's simplest form, "Why did Maxwell and Boltzman etc. link the energy of the imagined particles to temperature?"

The simplest answer is to imagine two chambers, filled with gas, and connected by a tube. The hotter one got that way because it received more energy, eg compression. They are at two different temperatures until the plug in the tube is removed. They come to the same temperature.

What cause temperature to change? The exchange of energy.

Part #1

"On a difficulty encountered in the theory of "black-body radiation"

“On a difficulty encountered in the theory of “black-body radiation”

We shall begin by taking the standpoint of Maxwell’s theory and the electron theory and the following case. Consider <u>a space enclosed</u> by completely reflecting walls containing a number of gas molecules and free electrons … that behave according to the theory of gases. Other electrons that are bound to points in space are resonators. They absorb and emit electromagnetic waves of definite periods. The resonators also are in equilibrium with the radiation fields.

Other treatments put the resonators in the walls.

What are the modes of radiation in the box? **By the way, I did my PhD with Prof. M W P Strandberg in his microwave lab. He was a grad student in WWII. He made and flew the first airborne radars.**

Since the fields go to zero at the walls there are 1, 2, 3, etc. wavelengths in each direction. So the allowed frequencies are nC/L for a finite box and a density of f(v) for an infinite one.

f (v) = $v\,\mathrm{d}v$ where v is the frequency

So here is the ultra-violet catastrophe:
The box has certain standing wave modes such that 1, 2, 3, etc. waves fit within the box. They have frequency

$$\nu = C/L$$

If each mode has an average energy of kT, then the total energy is kT times the sum of the integers from one to infinity. $\Sigma_n nkT$ = infinity

Or for the continuous case the integral of ν^2 from zero to infinity

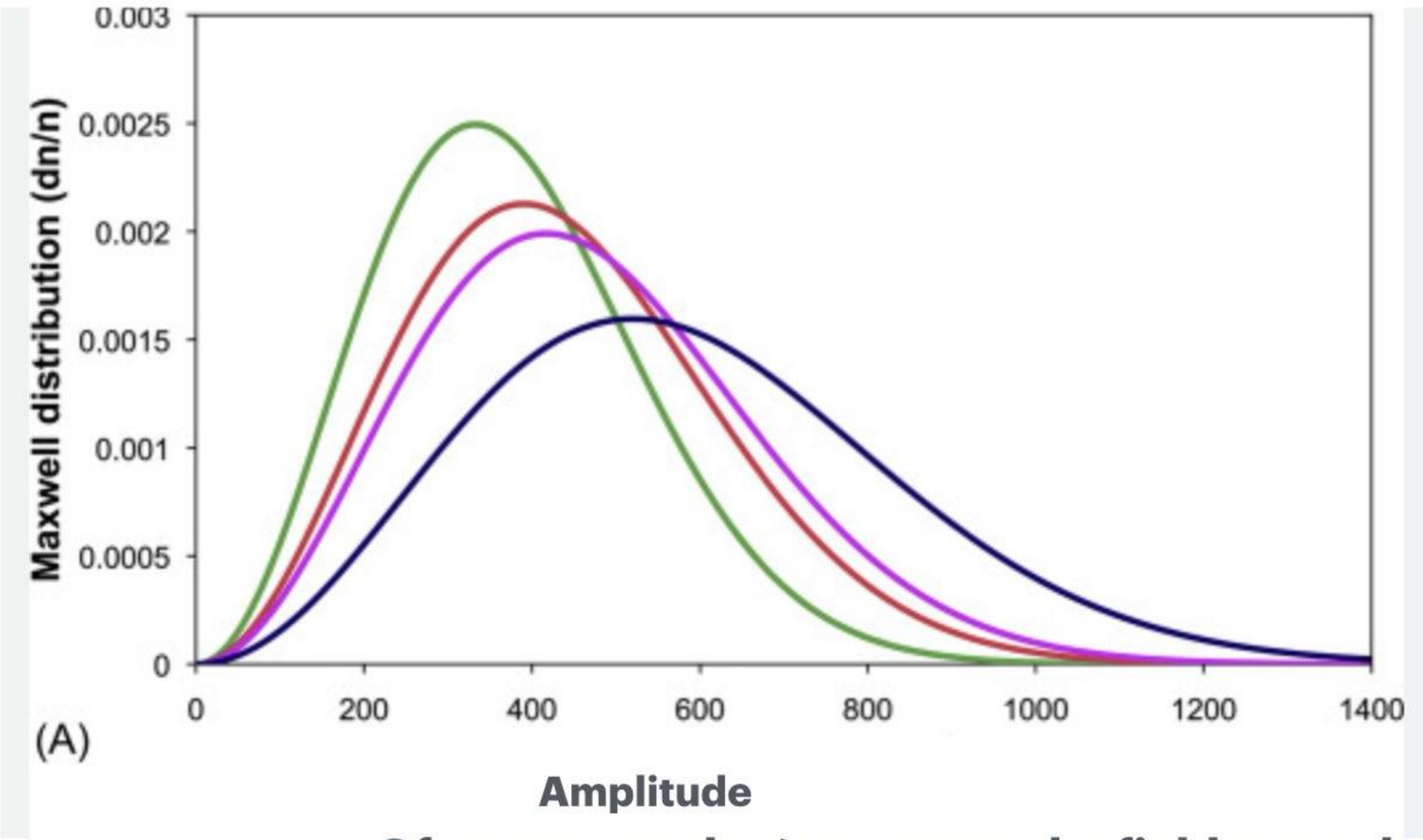
Let me repeat: It takes an infinite amount of energy to heat up this space!!

Part #2 On Planck's Determination of the elementary quanta

What is the problem?

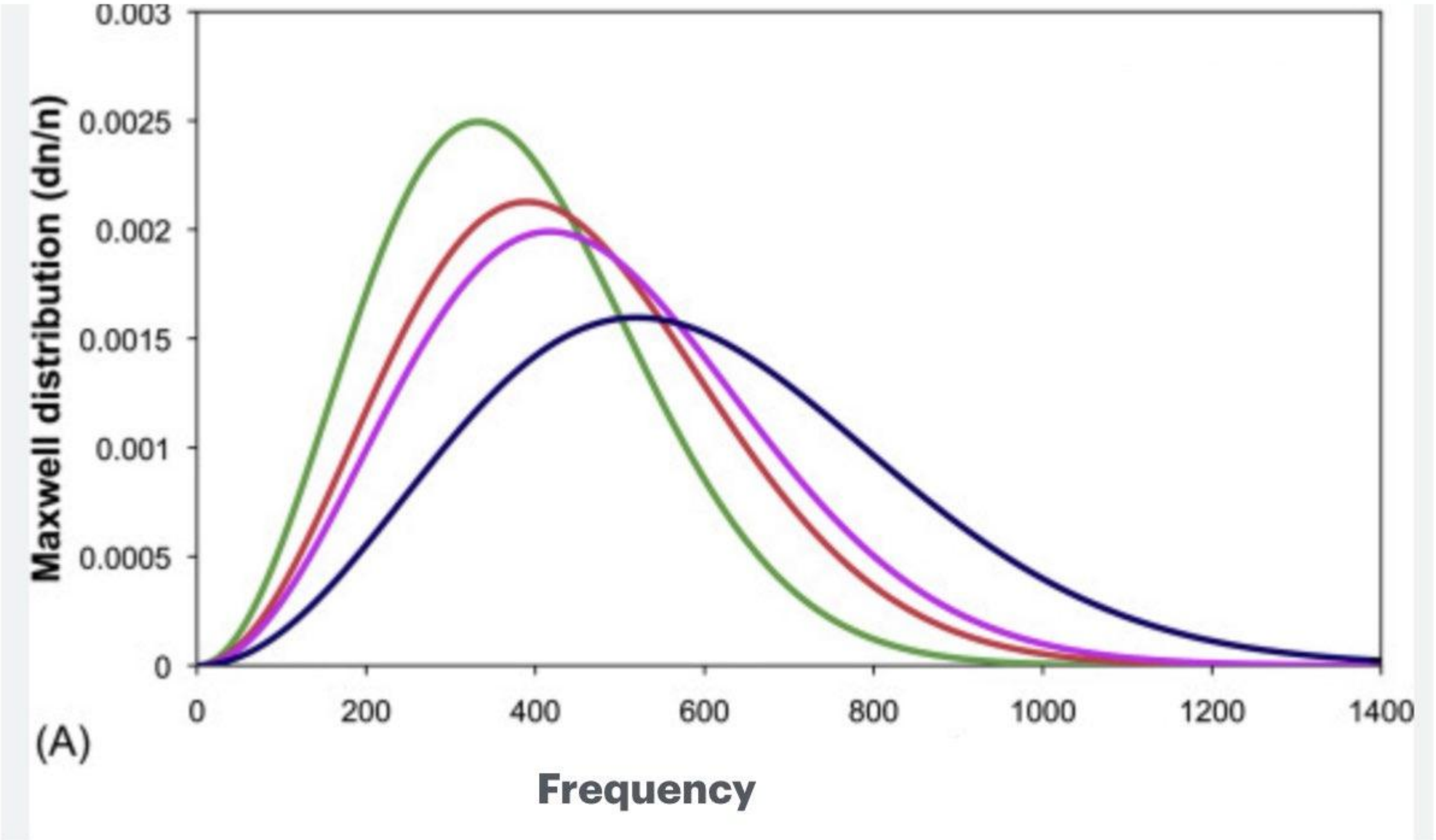
If we analyzed a wave of a given frequency and calculated its distribution of field strengths , assuming the strenght could take any value.

We might see something like the Maxwell distribution

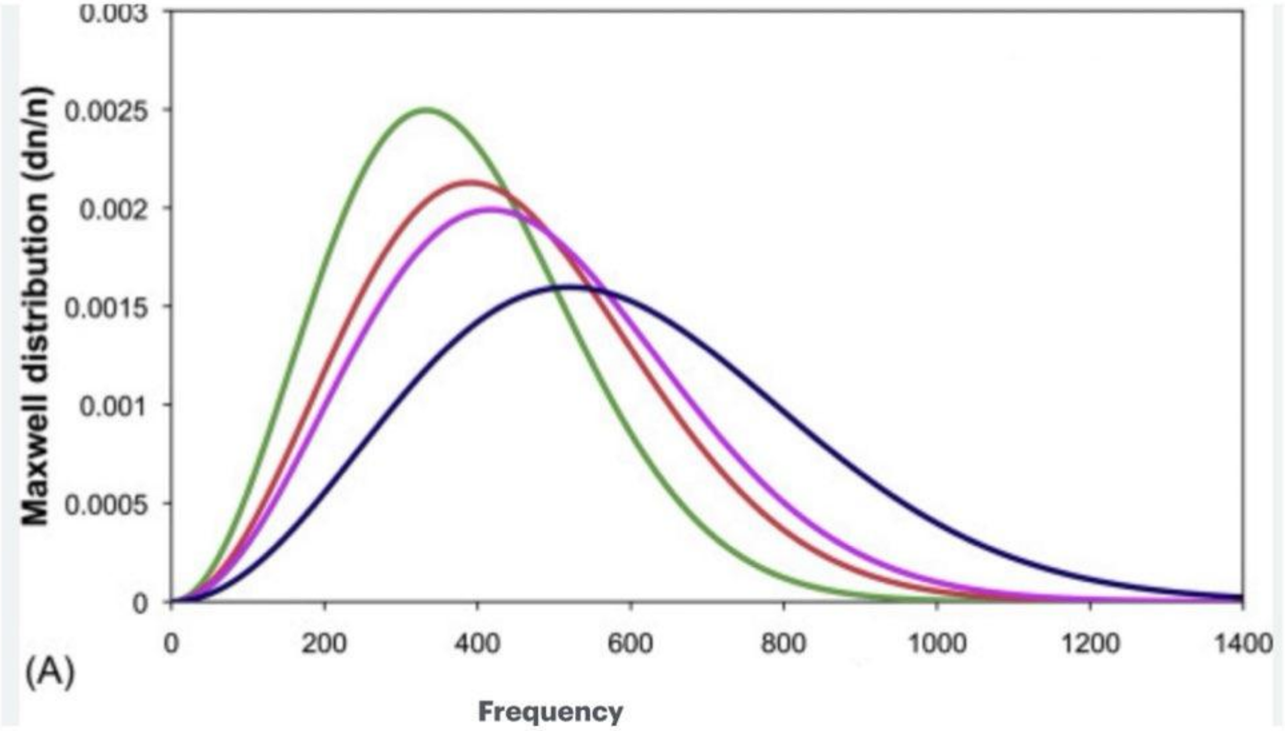


Of course we don't measure the <u>field</u> <u>strenght</u> at various times.

We do measure the average power at a given frequency.

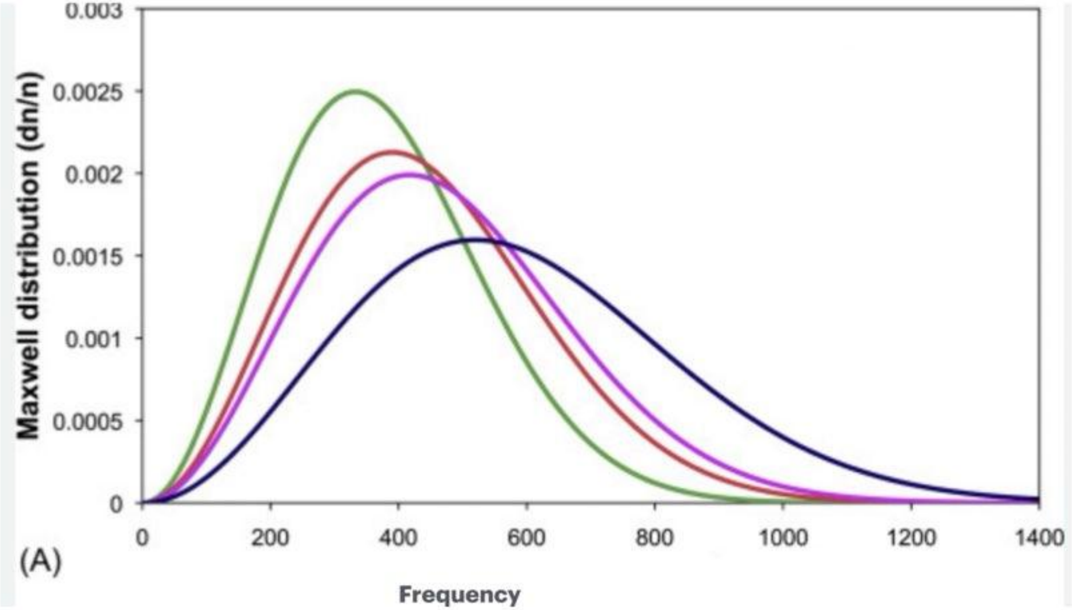


Ques: What do we actually see for the spectrum of light in the box?
Ans: Something that is similar to the Boltzman distribution, NOT WHAT WE PREDICT.



The density goes as v^2, so the low frequency observations are correct. But the missing high frequencies are the problem.

Is there something wrong with statistical mechanics?



The missing high frequencies are the problem.
Is there something wrong with statistical mechanics?

Planck's answer. No.
Something is wrong with assuming that the wave amplitudes, ie energies, can have any value.

Planck does makes two astounding assumptions:

1) Statistical mechanics is right. The probability of a mode having an energy is exp-(E/kT), if it does have an energy.

2) The mode can not have any energy. It can have energy in as multiples of some size. <u>That size must increase with frequency</u>. He picks the simplest, ie linear, relation <u>$E= h\nu$</u>

With these assumptions the the probability of a mode goes from $e^{-(E/kT)}dE$ where E is continuous to $e^{-(nh\nu/kT)}$ where n is an integer.

And

$$E_{avg} = \frac{\int E\, e^{(-E/kT)}}{\int e^{(-E/kT)}} \quad \text{Becomes} \quad E_{avg} = \frac{\Sigma_n (nh\nu)\, e^{-(nh\nu/kT)}}{\Sigma_n\, e^{-(nh\nu/kT)}}$$

You can do the sums. Let $X = e^{(-h\nu/kT)}$. Then the sums become

$$\frac{h\nu \sum_n n X^n}{\sum_n X^n}$$

The sums are:

$X+2X^2+3X^3 \ldots$ and

$1+X+X^2+X^3 \ldots$

The top series sums to $x(1-x)^{-2}$ and the bottom series to $(1-x)^{-1}$

The final result is the average energy for a frequency is

$$E_{avg} = h\nu/(e^{h\nu/kT} - 1)$$

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The Rayleigh-Jeans emission law with this as the average energy for a frequency matches the observed spectrum.

Raleigh- Jeans Law - The number of modes is $8\pi v^2/C^3$

In the continuous case the energy per mode is 1/2 kT

In the discreet case the energy is $hv/(e^{hv} - 1)$

The energy density is

$$U(v) = 8\pi v^2/C^3 \; [hv/(e^{hv/kT} - 1)]$$

Note: Density proportional to v^2 and T for low v and decreases as $e^{-hv/kT}$ for high v.

$$U(v) = 8\pi v^2/C^3\ [hv/(e^{hv/kT} - 1)]$$

The formula has both h and k. As I said in talk #1, Plank ascertained values for both. k became known as Boltzmann's constant and h as Plank's.

So where are we now. We have statistical mechanics which treats “physical” things like light waves as abstract math entities.
It gives the “right” results.

But can you actually observe any specific thing?

So where are we now. We have statistical mechanics which treats “physical” things like light waves as abstract math entities. It gives the “right” results.

But can you actually observe any specific thing?

OK! You can see the black body spectrum. But is there anything else you can actually “see”?

ξ3 On the entropy of radiation
ξ4 Limiting law for the entropy of monochromatic radiation at low density
ξ5 Molecular-theoretical investigation of the dependence of the entropy of gasses and dilute solutions on the volume
ξ6 Interpretation of the expression for the dependence of the entropy of monochromatic radiation on volume according to Boltzmann's principle.
ξ7 On Stokes' Rule

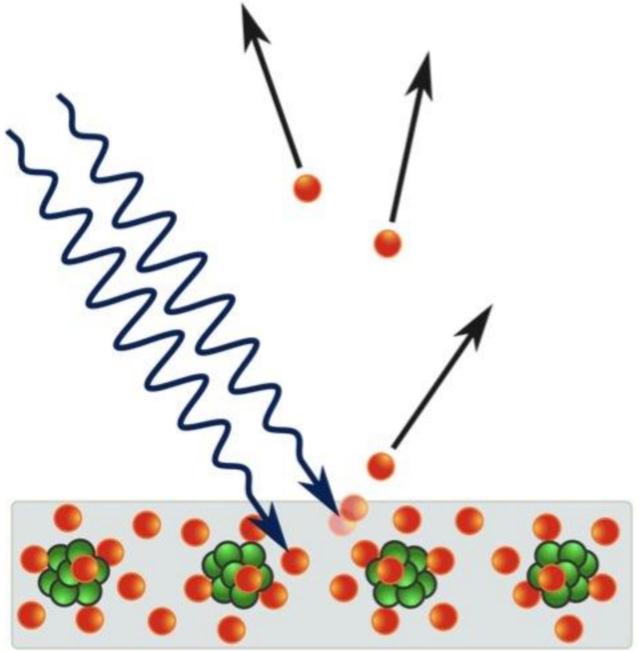
Comments:

ξ6 The conclusion is that if light waves behave like atoms, you get "the ultra-violet catastrophe".

ξ7 The conclusion is that photo-luminescence never combines the incoming energy to produce outgoing higher frequencies.

ξ8 On the generation of cathode rays by illumination of solid bodies

What Einstein did: The photo electric effect.

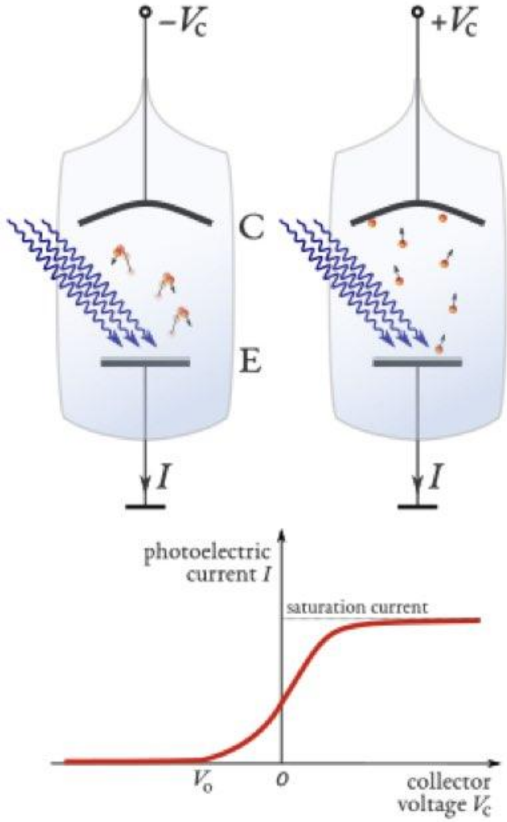


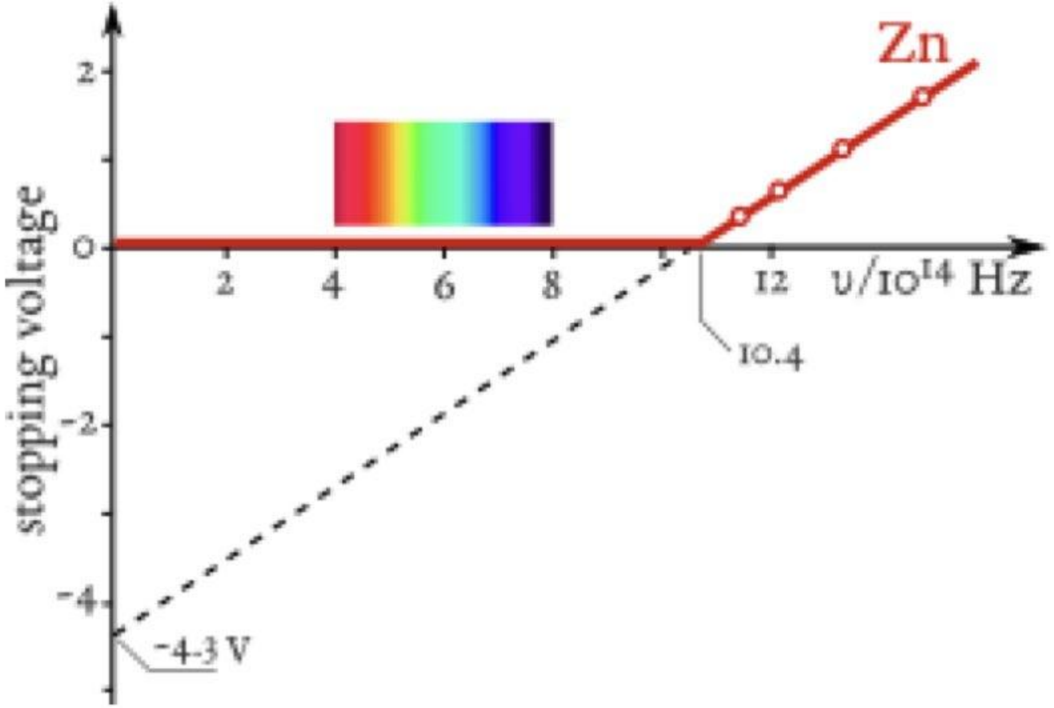
When UV light shines of a metal electrons pop out

Einstein's idea doesn't evolve complex math

Three observations:

1) The saturation current increases with UV intensity
2) No current unless the retarding voltage is greater than $-V_c$
3) The current starts at a lesser V_c with higher frequencies





No current if the retarding voltage is less than $-V_c$

No matter how intense the UV beam is and how long you wait no electrons pop out

The current starts at lower Vc for higher frequencies

At higher frequencies the electrons are popped out with more energy

Einstein concludes that UV light comes in small packets with energy hv. More intense UV light means more packets.

Einstein and Planck share the Nobel prize for photons.

That’s All Folks

Appendix: Why there isn't a two-photon process!! Einstein does address the question in regards to photoluminescence. He cites "Stokes Rule".

According to the conception of phenomena expanded, deviations from Stokes' rule are conceivable in the following cases:
1)When the number of simultaneous converting energy quanta a is so large that an energy quantum of light could obtain it energy from several producing quanta.
2)When the exciting light is produced by a body of such high temperature that Wien's law is no longer valid for the pertinent wavelength.