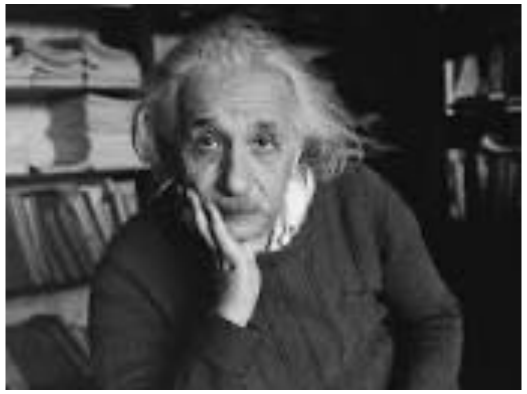
Talk #4 On the Electrodynamics of Moving Bodies



Special relativity part 2 - electrodynamics

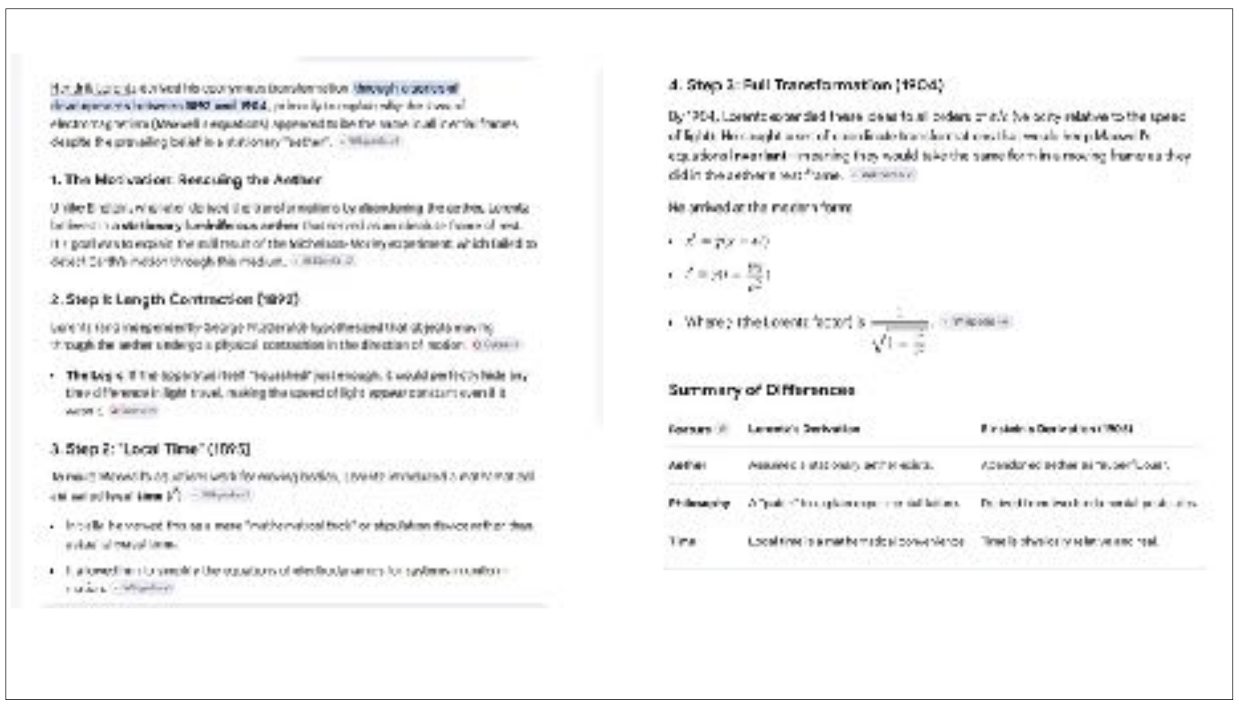
Review: Part 1 Special Relativity Kinematics

Einstien called attention to a magnet and a conductor and the difference in the interpretation, but not the answer, if you envisioned one or the other as moving.

He knew about the Michelson Morley experiment and proposed that the speed of light was a constant independent of the motion of the source. He also proposed that physical laws, such as the Maxwell equations, would be true in any system in constant motion. He wanted a fundamental/consistent theory of the relativity of constant motion.

He derived the Lorentz transform solely on two “postulates”. One was that the speed of light is a constant. The other was time and position were simply related. He used both the direction of the moving frame and a direction perpendicular to it to determine the two constants in the simple linear relation.

His result was the earlier Lorentz transform. But there was no “ether”, just a deep insight into the nature of time and space. The result was the same, but the theory was not.



Review: Part 1 Special Relativity Kinematics

His result was the slightly earlier Lorentz transform. <u>But</u> there was no "ether", just a deep insight into the nature of time and space. The result was the same, but the theory was <u>not</u>.

<u>So what?</u> 1)There is no such thing as simultaneous if anyone is moving!!!
2) If you move, someone else's distance shrinks and that someone else's time dilates.
2) You <u>absolutely</u> <u>can't</u> add velocities by putting multi-stages on rockets to get to speeds faster that light.

Was any of this testable? Nope!! Not in 1905!!

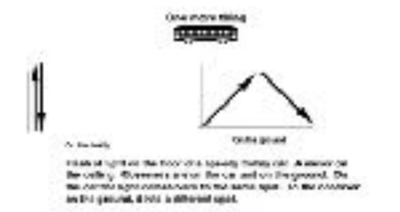
Special relativity means that satellites lose time, but general relativity means they gain it back from lesser gravity.

One more thing

On the trolly

On the ground

Flash of light on the floor of a speedy trolley car. A mirror on the ceiling. Observers are on the car and on the ground. On the car, the light comes back to the same spot. To the observer on the ground, it hits a different spot.



Einstein's derivation of the Lorentz transformation only considered the light going out for a time τ on the trolley. He addressed there being no such thing as simultaneity.

Therefore he did not address there being no such thing as co-location for moving systems.

He did say that if you got up and ran around you would be younger than if stayed at home.

He <u>focused</u> <u>on</u> <u>time</u> and didn't say anything about you being fatter if you ran around.

On to:

1) Maxwell’s equations
2) Maxwell’s equations with charges and currents
3)The energy of charges in electric fields

The second part - Electrodynamics

#6 Maxwell-Hertz equations transformation

#7 Doppler’s principle and aberration

#8 Energy of light rays and pressure on perfect mirrors

#9 Maxwell-Hetz transformation with current present

#10 Dynamics of the slowly accelerated electron

The so called Maxwell Equatuions

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$$

These equations relate the rate of spatial variations of the electric and magnetic fields (E and B) to the rate of their temporal changes.

J is current and ρ charge. Einstein considers individual charges, such as an electron. In this case J is ρv, where v is the charge's velocity. And more generally J= ∇*E v

Maxwell added the *d*E/*dt* term to Ampere's law

The first step is to write out Maxwell's equations in Cartesian coordinates

$(1/C)dE_x/dt = dB_z/dy - dB_y/dz$ And $(1/C)dB_x/dt = dE_y/dz - dE_z/dy$

$(1/C)dE_y/dt = dB_x/dz - dB_z/dx$ And $(1/C)dB_y/dt = dE_z/dx - dE_x/dz$

$(1/C)dE_z/dt = dB_y/dx - dB_x/dy$ And $(1/C)dB_z/dt = dE_x/dy - dE_y/dx$

The next step (or steps) is to apply the transforms to the six equations, remembering $\tau = \beta[t + (v/C^2)x]$ and $\xi = \beta[x + vt]$ and therefore

$d\ /dt = (d\ /d\tau)[d\tau/dt] + (d\ /d\xi)[d\xi/dt]$ And $d\ /dx = (d\ /d\tau)[d\tau/dx] + (d\ /d\xi)[d\xi/dx]$

The next steps are to substitute the d /dt and the d /dx. Then rearrange all the terms so the dE/dτ terms are on one side. The result is complicated. Two of the three components in each of the above six equations become two terms. And the two terms are the sum of an E field component and v/C times a B field component.

moving y is y', moving z is z' and x may be x' or ξ

For example, the three terms of the first become five terms

$(1/C)dE_x/dt = dB_z/dy - dB_y/dy$ Becomes

$(1/C)dE_x/d\tau = \beta d[B_z - (v/C)E_y]/dy' - \beta d[B_y + (v/C)E_z]/dz'$

But the Maxwell equations must be true in there simple form in the moving system. Therefore the above Maxwell equations are true if the coordinates and fields are the prime values of the moving system

x' y' z' and A'_x A'_y A'_z B'_x B'_y B'_z

The result is the relation between value of the fields in the moving system in terms of the stationary

$E'_x = E_x$ and $B'_x = B_x$
$E'_y = \beta[E_y-(v/C)B_z]$ and $B'_y = \beta[B_y+(v/C)E_z]$
$E'_z = \beta[E_z+(v/C)B_y]$ and $B'_z = \beta[B_z-(v/C)E_y]$

Which is $1/C\ d/E'_x d\tau = dB'_y/dz' - dB'_z/dy'$ for example

Einstein and wife must have loved algebra

Next Einstein examines the differences in the properties of a light ray as seen by a moving and a stationary observer. ϕ is the angle between the mover's velocity and the light ray.

He uses a slightly different notation and examines three features:

1) **The frequency of the light**
2) **The angle between the light ray's direction and the direction between the two observers.**
3) **The intensity of the light (*or energy density*)**

$\omega' = \omega\beta\,[1 - (v/C)\cos\phi]$ **Or** $\omega' = \omega\,[1-v/C]^{1/2}/[1+v/C]^{1/2}$ **if** $\phi=0$

$\cos\phi' = [\cos\phi - v/C]/[1-(v/C)\cos\phi]$

$A'^2 = A^2\beta[1 -(v/C)\cos\phi]^2$ **Or** $A'^2 = A^2\,[1-v/C]/[1+v/C\}$ **if** $\phi=0$

$$\omega = \omega' \frac{(1-v/c)^{1/2}}{(1+v/c)^{1/2}} \qquad A'^2 = A^2 \frac{(1-v/c)}{(1+v/c)}$$

Einstein makes a couple of comments:

First, this frequency effect is <u>not</u> the usual Doppler shift at all.
Second, the brightness of the light goes up as your velocity moving toward it increases. The light is infinitely bright if you could go at the speed of light.

Einstein's next step is to calculate the observed volume of a moving sphere.

$$S'/S = \frac{(1-v/c)^{1/2}}{(1-v/c)}$$

Remembering

$$A'^2 = A^2 \frac{(1-v/c)}{(1+v/c)}$$

You get for the contained energy

$$E'/E = \frac{(1-v/c)^{1/2}}{(1+v/c)^{1/2}}$$

Which is the same ratio as the frequencies. Of course!!!!!

Even if the energy intensity and density are different, how about the light's pressure?

Einstein next looks at the reflection from a moving mirror. He starts with the light in the stationary frame. He transforms into the moving frame, then reflects so the light is going in the return direction. And finally transforms back to the stationary frame.

For direct reflection the result is nothing appears different in the stationary frame even though the light that is going out and back in the moving frame is different.
The the energy and momentum are reflected as if the mirror were not moving. And therefore the pressure on the mirror is the same.

If the light is not directly incident the light pressure is
$P=(2A^2/8\pi)\ \beta^2[\cos\phi - v/c]^2$ Or for small v/c $P=(2A^2/8\pi)\ (\cos\phi)^2$

Let's look at The Doppler effect.
The guy on the plane lets out beeps at τ_1, τ_2, τ_3 etc.
She "hears" the first beep at τ_1+r/c. The second occurs when the plane is closer to her. So she hears it at $\tau_2+[r-v(\tau_2-\tau_1)]/c$

His difference on the plane is $(\tau_2-\tau_1)$.
She hears a high frequency.
Her difference, (t_2-t_1), is $(\tau_2-\tau_1)(1-v/c)$.

This is the usual Doppler effect! Is it correct?

<u>Nope</u>!! Her and his clocks do not run at the same rate.
A "t" increment is not the same as a "τ" increment.
You have to transform the events correctly!!

Let's add charges and currents

The results for frequency and intensity were based on the equations where there were no currents, or rather charges, present. Now let's put an electron in the picture.

The equations become

$(1/C)\{v_x\rho + dE_x/dt\} = dB_z/dy - dB_y/dz$ $(1/C)dB_x/dt = dE_y/dz - dE_x/dy$

$1/C)\{v_y\rho + dE_y/dt\} = dB_x/dz - dB_z/dx$ $(1/C)dB_y/dt = dE_z/dx - dE_x/dz$

$(1/C)\{v_z\rho + dE_z/dt\} = dB_y/dx - dB_x/dy$ $(1/C)dB_z/dt = dE_x/dy - dE_y/dx$

The key differences are the three components of the velocity v_x, v_y, v_z and, perhaps, ρ

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}\right)$$

ρ is the charge, so the components of the current, J, are ρv_x, ρv_y and ρv_z
But the divergence of E is ρ. Therefore

$\rho = dE_x/d_x + dE_y/d_y + dE_z/d_z$ and

$J_x = v_x(dE_x/d_x + dE_y/d_y + dE_z/d_z)$
$J_y = v_y(dE_x/d_x + dE_y/d_y + dE_z/d_z)$
$J_z = v_z(dE_x/d_x + dE_y/d_y + dE_z/d_z)$

Einstein examines the transformation of J, ie ρ and v. He looks at a moving charge. He concludes that the difference in the observed velocities explains the differences in currents and that the charge is the same for both observers.

The transformations are bunch of algebra

I have been cavalier about ε_0 and μ_0

Einstein next looks at a "point particle" at rest that is then exposed to an electric field. Initially

$$md^2X_{x,y,z}/dt^2 = \varepsilon E_{x,y,z}$$

Consider a frame that moves along with the changing position/velocity of the electron. Pick the direction of motion to be the x direction. The above equation is the same in the new frame.

$md^2X'_{x',y',z'}/d\tau^2 = \varepsilon E'_{x',y',z'}$ but we know relation between the fields in the old (stationary) frame and the moving frame.

$\tau=\beta(t-v/c^2x)$ and $d/d\tau = (1/\beta)(d/dt)$

$$md^2x/dt^2 = \varepsilon(1/\beta^3)E_x$$
$$md^2y/dt^2 = \varepsilon(1/\beta)[E_y-(v/c)B_z]$$
$$md^2z/dt^2 = \varepsilon(1/\beta)[E_z-(v/c)B_y]$$

Or

$$\beta^3 md^2x/dt^2 = \varepsilon E'_x$$
$$\beta^2 md^2y/dt^2 = \varepsilon E'_y$$
$$\beta^2 md^2z/dt^2 = \varepsilon E'_z$$

$\beta=1/[1-(v/c)^2]$

The second form of the equations says that a moving electric field applied to a moving electron accelerates it.

BUT that field accelerates it as if its mass is different. AND that apparent difference depends on whether you are trying to push it in the direction it is going or bend its path.

It is worth repeating the equations of motion of an electron in an electric field applied in the primed system:

$$m\beta^3(d^2x/dt^2) = \varepsilon E_x = \varepsilon E'_x$$
$$m\beta^2(d^2y/dt^2) = \varepsilon\beta[E_y - v/cB_z] = \varepsilon E'_y$$
$$m\beta^2(d^2z/dt^2) = \varepsilon\beta[E_z - v/cB_x] = \varepsilon E'_z$$

Note: The first equation, ie in the direction of motion, is just "the force equal mass times acceleration". BUT the mass increase as β^3.

$$m\beta^3(d^2x/dt^2) = \varepsilon E_x = \varepsilon E'_x$$

Einstein does two more things relating to the relative deflection and radius of curvature of electric and magnetic fields applied to moving charges/electrons.

Einstein next looks at the change of kinetic energy that a moving charges acquires in an electric field. The work the field does is

$$W = \int \varepsilon E_x dx = m\int_0^V \beta^3 v dv$$

since $\beta=1/[1-(v/c)^2]^{1/2}$ the integration is though, but doable

$W = mc^2\{[1/(1-(v/c)^2]^{1/2} - 1\}$ and W = Potential energy gain

$$W = mC^2\{\beta - 1\}$$

The change from integral in space to integral in velocity

$$\int \varepsilon E_x \, dx = \int m\beta^3 \, [d(dx/dt)dt] \, dx = \int_0^V m\beta^3 \, v dv$$

The integral of
(dv/dt)dx over a range of space
is the same as
(v)dv over the range of velocity

(dv/dt)dx versus (dx/dt)dv

$\int \beta^3(v)(dv/dt)dx$ and <u>x=vt</u> Therefore dx=vdt and

(dv/dt)dx = (dv/dt)(vdt) = vdv

Does the inertia of a body depend on its energy content?

The last paper

The last paper:
Does the Inertia of a Body Depend Upon Its Energy Content?

Let a system of plane waves, traveling in the x direction, possess an energy E. The energy of these waves in a system moving parallel to it is

$$E^* = E \frac{1 - v/c}{[1-(v/c)^2]^{1/2}}$$

Let a body at rest simultaneously emit two amounts of light L/2; in opposite directions. The energy before equals the energy after

$$E_0 = E_1 + L/2 + L/2$$

To a moving observer the relation is

$$H_0 = H_1 + L/2 \frac{1 - v/c}{[1-(v/c)^2]^{1/2}} + L/2 \frac{1 + v/c}{[1-(v/c)^2]^{1/2}} = H_1 + \frac{L}{[1-(v/c)^2]^{1/2}}$$

The last paper: Does the Inertia of a Body Depend Upon Its Energy Content?

Let a body at rest simultaneously emit two amounts of light L/2;
in opposite directions.

Is this possible? Does Einstein know of suspect it isn't?

$$E_0 = E_1 + L/2 + L/2$$
$$H_0 = H_1 + L/[1-(v/c)^2]^{1/2}$$

Subtracting these two gives

$$(H_0 - E_0) - (H_1 - E_1) = L\{[1/[1-(v/c)^2]^{1/2} - 1\}$$

Einstein next argues that H-E are the energy values of the same body, but viewed in two different systems. Hence they are the reduction of kinetic energy.

Einstein notes that the change depends only on L and the observer's velocity, not on any changing property of the emitting body.

The formula looks like the energy change of a body in an E field.

$$W = mc^2\{[1/(1-(v/c)^2]^{1/2} - 1\}$$

1} The emission of light changes the kinetic energy.
2} The change did not depend on the properties of the emitting particle.
3} The formula for the change has the same velocity dependence as the work done by an electric field, increasing the kinetic energy of a particle.

Einstein's Genius - A wild surmise

$$E/C^2=\Delta M$$

He imagined an experiment that didn't (can't?) exist and got the amazing answer!

"If a body releases energy L in the form of radiation, its mass decreases by L/C^2.
The mass of a body is a measure of its energy content: if the energy changes by L, the mass changes by $L/(9*10^{20})$, if the energy is measured in ergs and the mass in grams.
Perhaps it will prove possible to test this theory using bodies whose energy content is variable in a high degree (e.g. salts of radium)."

A. Einstein. Bern, September 1905

In 1905 Einstein said
You guys who are talking about ether drag, etc., “forget it!”

Unlike every other wave, the speed of light is an absolute constant!
Space and time are not what you think they are!
But the laws, both mechanical and electrical, are the same!
But the high speed <u>mechanical</u> ones are not as you think they are!
eg An electron in a field doesn’t increase its speed linearly without end!

Imagine a phenomenon that you probably won’t ever see!
Two flashes of luminescence leave an atom in opposite directions.
That demonstrates that mass and energy are the same thing!!

How About That!

The man had chutzpah