

## Measurement vs. Hadamard Gate

*QUESTION: I start with  $|+z\rangle$  and after selecting the  $|+x\rangle$  state I'm in a coherent superposition of plus and minus  $z$  states. So why isn't this coherent superposition enough to consider the whole operation to be a gate?*

The issue is the difference between **preparation via measurement** and a **unitary gate (like Hadamard)**.

### Your reasoning (so far)

You said:

1. Start in  $|+z\rangle$
2. Measure along  $x$  (Stern–Gerlach in  $x$ -direction)
3. Select the  $|+x\rangle$  outcome

And you note:

$$|+x\rangle = \frac{1}{\sqrt{2}}(|+z\rangle + |-z\rangle)$$

So yes—you now *have* a coherent superposition in the  $z$ -basis.

### So why isn't this a Hadamard?

Because of what happened to the state during measurement.

### Key distinction

#### 1) Measurement-based preparation

Your process:

- You measure in the  $x$ -basis
- The state collapses probabilistically:

$$- 50\% \rightarrow |+x\rangle$$

– 50%  $\rightarrow$   $| - x \rangle$

Then you:

- discard the  $| - x \rangle$  cases

This is **post-selection**.

## 2) Hadamard gate (unitary)

A Hadamard does:

$$| + z \rangle \rightarrow \frac{1}{\sqrt{2}}(| + z \rangle + | - z \rangle)$$

But crucially:

- Deterministic
- Reversible
- No measurement
- Preserves phase relationships for all inputs

## What measurement destroys

When you measure:

- You lose coherence with respect to the original state

Even if you end up in  $| + x \rangle$ , the process:

- Introduced randomness
- Destroyed phase information about prior superpositions

## The deeper issue

A Hadamard must work on **any input state**, not just  $| + z \rangle$ .

## Example

Suppose input is:

$$\frac{1}{\sqrt{2}}(|+z\rangle + |-z\rangle)$$

A Hadamard gives:

$$|+z\rangle$$

But your measurement-based method:

- Measures in  $x$ -basis
- Randomly collapses to  $|+x\rangle$  or  $|-x\rangle$

You do not recover the correct output.

## Big insight

- Measurement is not a linear operation
- Quantum gates must be linear (unitary)

That's the core difference.

## Physical analogy

Your Stern–Gerlach idea is like:

- Splitting beam  $\rightarrow$  measuring  $\rightarrow$  keeping one branch

This is:

- State preparation
- Not a coherent transformation

## What a real “Hadamard-like” device would be

Instead of measurement, you need something like:

- A magnetic field causing spin precession

For example:

$$H \propto \sigma_y$$

Time evolution:

$$U = e^{-iHt} = R_y(\theta)$$

This rotates the spin **coherently**, without collapse.

## Stern–Gerlach vs unitary evolution

Process	Nature	Result
Stern–Gerlach measurement	non-unitary	collapse
Magnetic field rotation	unitary	coherent rotation

## Bottom line

- Your method does produce  $|+x\rangle$
- But via measurement + post-selection
- Not equivalent to a Hadamard gate

A Hadamard must:

- Act deterministically
- Preserve coherence
- Work on all input states

## One-sentence intuition

Measurement creates a superposition by collapse; a gate rotates into a superposition coherently.