Quantum Computing – Hello World

Bill Celmaster

April 28, 2026

Overview

- Brief review of 2-basis-state quantum mechanics
- Brief review of product states
- Gates (pictures, words), unitary matrices and quantum circuits
- A simple quantum circuit and algorithm for IBM's computer
  - Introduction to IBM's environment
  - Some Python scripts (for non-Python people)
  - Translating those scripts to pictures and gates
  - Predicting the quantum behavior of the quantum circuit
  - Execution of the algorithm on actual hardware

1-**Qbit** systems (2D)

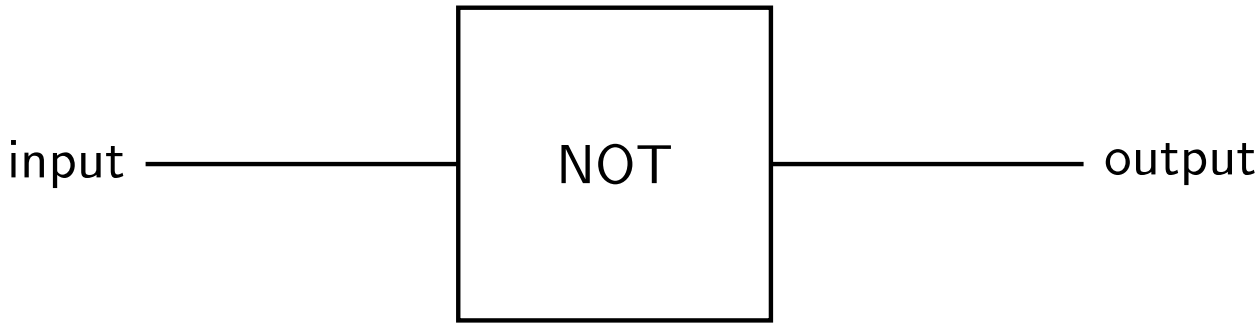
- Orthogonal basis states are generically labeled $|0\rangle$ and $|1\rangle$.
- Example: electron-spin in z-direction: spin up $\uparrow$, spin down $\downarrow$
  - Spin states are denoted $|\uparrow\rangle$ and $|\downarrow\rangle$
- General (normalized) states are superpositions of basis states
  - $|Q\rangle = \alpha|0\rangle + \beta|1\rangle$
    - α, β are complex numbers
    - $|\alpha|^2 + |\beta|^2 = 1$
- Vector notation: $|\mathbf{Q}\rangle = \alpha|0\rangle + \beta|1\rangle \rightarrow \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$
- For future convenience, we'll refer to $|0\rangle$ and $|1\rangle$ as the **Z** eigenstates

Qbit inner products and measurement

- We define $\langle \mathbf{Q}' | \mathbf{Q} \rangle$ as $(\alpha')^* \alpha + (\beta')^* \beta$
- Vector notation: $\langle \mathbf{Q}' | \mathbf{Q} \rangle \rightarrow (\mathbf{Q}', \mathbf{Q}) \equiv \mathbf{Q}'^{\dagger} \mathbf{Q} = \begin{pmatrix} \alpha'^* & \beta'^* \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$
- In principal, there are measuring devices $O_{|s\rangle}$ such that
  - They report "yes" when the system is in state $|s\rangle$
  - After they report "yes", the system is still in state $|s\rangle$
- If a system is in $|\mathbf{Q}\rangle$, then
  - The probability that $O_{|0\rangle}$ is "yes", is $|\alpha|^2$
  - The probability that $O_{|1\rangle}$ is "yes", is $|\beta|^2$
  - The probability that $O_{|\mathbf{Q}'\rangle}$ is "yes" is $|\langle \mathbf{Q}' | \mathbf{Q} \rangle|^2$

Gates: Boolean and quantum gates

- Digital circuits consist of sequences of gates.
  - Each gate takes M inputs and N outputs
- An example of a classical (Boolean) gate



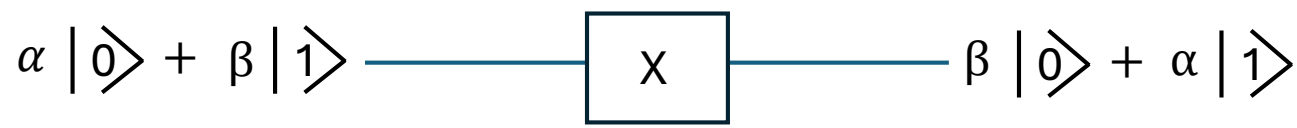
with truth table

input	**output**
0	1
1	0

- An example of a quantum gate

Single Qbit gate and matrix notation

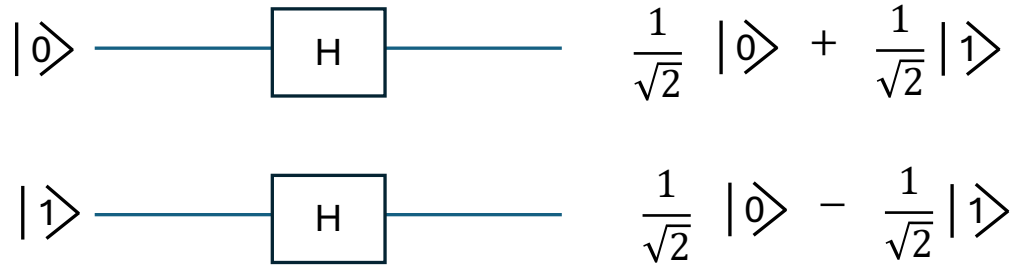
- Input is $|\mathbf{Q}\rangle$, output is $|\mathbf{Q}'\rangle$. Both states are normalized. **U** is linear.
- Matrix notation: $\mathbf{Q}' = \mathbf{UQ}$ where **U** is unitary
- Example: If $|\mathbf{Q}\rangle = |0\rangle$, then $|\mathbf{Q}'\rangle = |1\rangle$; if $|\mathbf{Q}\rangle = |1\rangle$, then $|\mathbf{Q}'\rangle = |0\rangle$.
  - We call this "flipping the Qbit".
  - Linearity:



- Matrix notation for the example. $\begin{pmatrix}\beta\\ \alpha\end{pmatrix} = \mathbf{X}\begin{pmatrix}\alpha\\ \beta\end{pmatrix}$ **or** $\mathbf{X} = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$.

Hadamard gate and Measurement

- Using circuit-pictures:
  - **Hadamard gate**



  - **Measurement**

    By default, we measure the operator **Z**

    The probability of measuring "0" is $|\alpha|^2$, prob. of "1" is $|\beta|^2$.

- Using vectors and operators:
  - **Hadamard**: Generates Qbit $\mathbf{Q}' = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$
  - **M**: $P(0) = \frac{1}{2} | \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} |^2$

    $P(1) = \frac{1}{2} | \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} |^2$

A real-world Hadamard gate

Reference material: "Quantum computing with photons" by Stefanie Barz (Oxford)

- One implementation of Qbits is to use light (photon) polarization.
- Horizontal: $|H\rangle \to |0\rangle$, Vertical: $|V\rangle \to |1\rangle$.
- Use a uniaxial birefringent (e.g. quartz) wave plate.

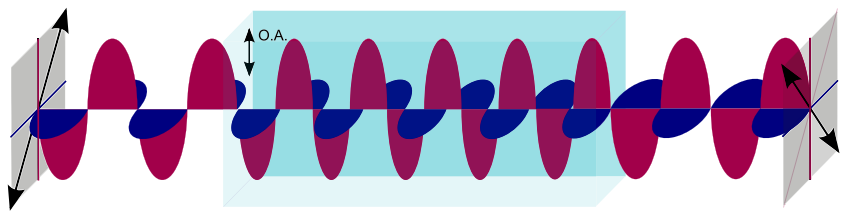


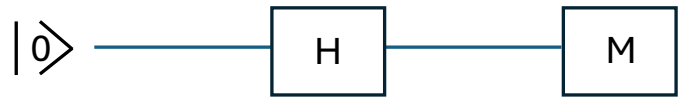
Figure: Half-wave plate. Different refractive indices for vertical and horizontal polarizations.

Here, we start with $\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$ and end with $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

- Rotate crystal by θ from the horizontal axis.
  $U(\theta) = \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$
- E.g. $\theta = \frac{\pi}{8}$ is the Hadamard gate.

The "Hello World" circuit

- Circuit-picture:



- Matrix-vector:

$$P(0) = |\begin{pmatrix}1 & 0\end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} \begin{pmatrix}1\\ 0\end{pmatrix}|^2 = \frac{1}{2}$$

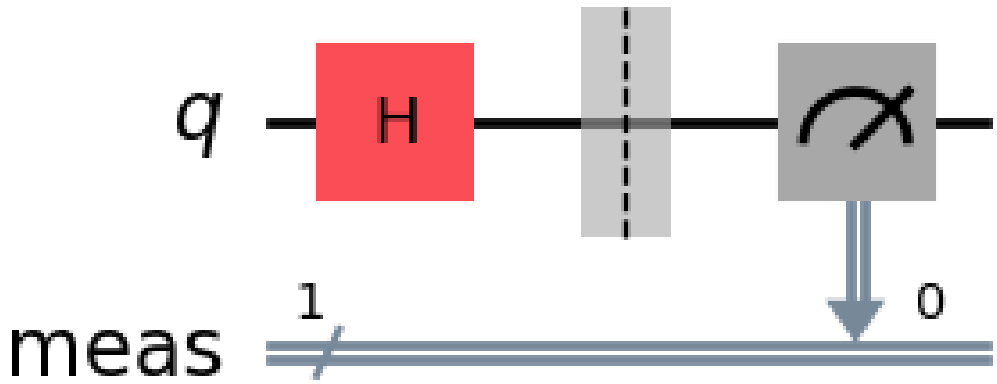
$$P(1) = |\begin{pmatrix}0 & 1\end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} \begin{pmatrix}1\\ 0\end{pmatrix}|^2 = \frac{1}{2}$$

- What would an experiment look like?
  - Start with Qbit $|0\rangle$.
  - Transform it to the Bell state $\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$
  - Measure the **Z** value (e.g. z-spin "up" or "down")
  - Repeat 100 times
  - Report fraction of 0's and fraction of 1's.

The "Hello World" Python quantum-simulator program

```
from qiskit import QuantumCircuit
from qiskit.primitives import StatevectorSampler
from IPython.display import display

# 1. Create a 1-qubit circuit
qc = QuantumCircuit(1)

# 2. Apply Hadamard gate
qc.h(0)

# 3. Measure
qc.measure_all()

#3c. Display this meta-circuit
display(qc.draw("mpl"))

# 4. Create Sampler
sampler = StatevectorSampler()

# 6. Run the circuit with the number of shots
job = sampler.run([qc], shots=100)

# 7. Get results
result = job.result()
meas = result[0].data.meas

# 8. Convert to counts
counts = meas.get_counts()


print(counts)

display(qc.draw("mpl"))
```

Output



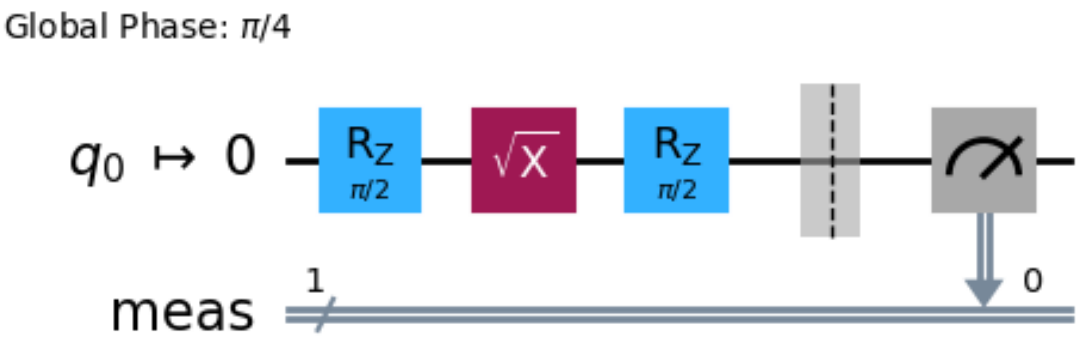
{'0': 57, '1': 43}

The "Hello World" program for quantum hardware

```
##                          Import definitions of functions
from qiskit import QuantumCircuit
from qiskit_ibm_runtime import QiskitRuntimeService
from qiskit.transpiler.preset_passmanagers import generate_preset_pass_manager
from qiskit_ibm_runtime import Sampler
from IPython.display import display

##         First create a circuit locally
#                  1. Create a 1-qubit circuit
qc = QuantumCircuit(1)
#                  2. Apply Hadamard gate
qc.h(0)
#                  3. Measure
qc.measure_all()

##         Set up the connection to the appropriate IBM cloud quantum computer
#                  4. Loads my saved IBM credentials and connects to IBM cloud
service = QiskitRuntimeService()
#                  5. Runs locally, queries IBM cloud for least busy devices, and
#                                          returns a handle for the remote backend
backend = service.least_busy(simulator=False, operational=True)

##         Transpiler step: locally convert my circuit to use real quantum hw -- gates and topology
#                  6. Convert to an ISA circuit and layout-mapped observables.
pm = generate_preset_pass_manager(backend=backend, optimization_level=1)
isa_circuit = pm.run(qc)
#                  7. Show the new circuit
display(isa_circuit.draw("mpl", idle_wires=False))
```

"Hello World" for quantum hardware, cont'd

```
##         Running and analyzing the circuit
#                 8. Create the program to be run on the IBM cloud, which analyzes
#                                         outputs of quantum circuit
sampler = Sampler(mode=backend)
#                 9. Run the circuit with the number of shots
job = sampler.run([isa_circuit], shots=100)
print("Job ID:", job.job_id())
#                 10. Get results (waits until job finishes)
result = job.result()
#                 11. Extract counts
counts = result[0].data.meas.get_counts()
print("Counts:", counts)
```




```
Job ID: d6vnhg2tnsts73ete5a0
Counts: {'0': 45, '1': 55}
```

_____ *(a second run, but without the diagram)*

```
Job ID: d6vnm0s69uic73cjgq3g
Counts: {'1': 52, '0': 48}
```

Interpretation of the transpiled circuit

- Instead of the $\mathbf{H}$ gate, the circuit now has 3 sequential gates: $\mathbf{R_z}\left(\frac{\pi}{2}\right)$, $\sqrt{\mathbf{X}}$, $\mathbf{R_z}\left(\frac{\pi}{2}\right)$
- $\mathbf{R_z}\left(\frac{\pi}{2}\right) = \begin{pmatrix} e^{-i\frac{\pi}{4}} & 0 \\ 0 & e^{i\frac{\pi}{4}} \end{pmatrix}$ and $\sqrt{\mathbf{X}} = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{i\frac{\pi}{4}} & e^{-i\frac{\pi}{4}} \\ e^{-i\frac{\pi}{4}} & e^{i\frac{\pi}{4}} \end{pmatrix}$
  - $\mathbf{R_z}\left(\frac{\pi}{2}\right)$ represents a spin rotation by $\frac{\pi}{2}$ around the z-axis.
  - $\sqrt{\mathbf{X}}^2 = \mathbf{X}$.
- These gates can be implemented in hardware.
- Multiply the matrices. $\mathbf{R_z}\left(\frac{\pi}{2}\right)\sqrt{\mathbf{X}}\mathbf{R_z}\left(\frac{\pi}{2}\right) = e^{-i\frac{\pi}{4}}\mathbf{H}$.
- Recall for the original "hello world" circuit, that $P_{\text{orig}}(0) = |\begin{pmatrix} 1 & 0 \end{pmatrix}\mathbf{H}\begin{pmatrix} 1 \\ 0 \end{pmatrix}|^2 = \frac{1}{2}$, etc.
- Similarly, for the transpiled circuit, we have $P_{\text{orig}}(0) = |\begin{pmatrix} 1 & 0 \end{pmatrix}\mathbf{R_z}\left(\frac{\pi}{2}\right)\sqrt{\mathbf{X}}\mathbf{R_z}\left(\frac{\pi}{2}\right)\begin{pmatrix} 1 \\ 0 \end{pmatrix}|^2 =$ $|\begin{pmatrix} 1 & 0 \end{pmatrix}e^{-i\frac{\pi}{4}}\mathbf{H}\begin{pmatrix} 1 \\ 0 \end{pmatrix}|^2 = |\begin{pmatrix} 1 & 0 \end{pmatrix}\mathbf{H}\begin{pmatrix} 1 \\ 0 \end{pmatrix}|^2 = \frac{1}{2}$, etc.