

Quantum Computing – Quantum Teleportation

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Overview

- ▶ Direct transmission of quantum state from A to B
 - ▶ Source
 - ▶ Optical fiber
 - ▶ Satellite
- ▶ Storage of quantum state at B
- ▶ Teleportation of the EPR state
 - ▶ Using a repeater
 - ▶ Entanglement swapping
 - ▶ Using a beam-splitter to measure some Bell states
 - ▶ Non-linear optics for remaining Bell states
 - ▶ Communication of the measurement result
 - ▶ Conversion of distant Bell state into a distant EPR state
 - ▶ Entanglement swapping overcomes transmission-loss
- ▶ Teleportation of arbitrary states with help of stored EPR pairs
 - ▶ Overcomes transmission-loss of arbitrary states

Direct transmission of quantum states

- ▶ Source at A
 - ▶ Non-photonic quantum state (e.g. trapped ion)
 - ▶ Converted to photon quantum state for transmission
 - ▶ One-photon quantum state or entangled EPR pair
 - ▶ Often created by a laser beam incident on crystal
 - ▶ Process is known as SPDC (Spontaneous Parametric Down-Conversion)
- ▶ Optical fiber
 - ▶ Typically glass core surrounded by cladding
 - ▶ Photon confined by total reflection; polarization is preserved
 - ▶ Some sources of degradation
 - ▶ At $\lambda = 1550\text{nm}$, attenuation by glass-absorption $\approx 0.2\text{dB/km}$ (99% loss at 100 km; $P \approx 10^{-0.02L}$)
 - ▶ Scattering due to imperfections leads to more loss
 - ▶ Anisotropy of cable leads to loss of state-fidelity
- ▶ Satellite
 - ▶ Photons are transmitted and received by telescopes
 - ▶ The main source of degradation is diffraction (spreading)
 - ▶ Reduce this with larger telescope apertures
 - ▶ $P \propto L^{-2}$
 - ▶ With 1m apertures, $P_{\text{theor}} > 0.9$ at 100 km

Storage of a quantum state

- ▶ Teleportation of $|\psi\rangle$ requires a separate previously entangled state.
 - ▶ $|\psi\rangle = \frac{1}{\sqrt{2}} (|H\rangle_A |H\rangle_B + |V\rangle_A |V\rangle_B)$
- ▶ So we need a way of storing that state
- ▶ Set up the photon to be absorbed coherently by some material
 - ▶ $|H\rangle_{\text{photon}} \rightarrow |M_H\rangle$ (Horizontal polarization)
 - ▶ $|V\rangle_{\text{photon}} \rightarrow |M_V\rangle$ (Vertical polarization)
- ▶ Then $\frac{1}{\sqrt{2}} (|H\rangle_A |H\rangle_B + |V\rangle_A |V\rangle_B) \rightarrow \frac{1}{\sqrt{2}} (|H\rangle_A |M_H\rangle + |V\rangle_A |M_V\rangle)$
- ▶ On the A side, store the qubit the same way
- ▶ Later we want to reconstruct the entangle photon
 - ▶ Shoot laser at the material to stimulate reverse process
- ▶ In our analysis, assume EPR qubits are stored

Teleportation of the EPR state – Part 1

- ▶ Using a repeater B, between A and C
 - ▶ Entangled qubits via transmission

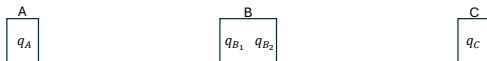


Figure: Two EPR pairs of qubits – $q_A q_{B_1}$ & $q_{B_2} q_C$

- ▶ $|\Psi\rangle$ is a 4-qubit state which is a product of 2-qubit states
 - ▶ Define $|\Phi^+\rangle = |EPR\rangle = \frac{1}{\sqrt{2}} (|HH\rangle + |VV\rangle)$
 - ▶ Then $|\Psi\rangle = |\Phi^+\rangle_{AB_1} \otimes |\Phi^+\rangle_{B_2C}$
- ▶ Rewrite 2-qubit states using a new 2-qubit basis – the Bell basis
 - ▶ $|\Phi^\pm\rangle = \frac{1}{\sqrt{2}} (|HH\rangle \pm |VV\rangle)$
 - ▶ $|\Psi^\pm\rangle = \frac{1}{\sqrt{2}} (|HV\rangle \pm |VH\rangle)$
- ▶ Neat trick:

$$\begin{aligned}
 |\Psi\rangle &= \frac{1}{2} (|H\rangle_A \otimes |H\rangle_{B_1} + |V\rangle_A \otimes |V\rangle_{B_1}) \otimes (|H\rangle_{B_2} \otimes |H\rangle_C + |V\rangle_{B_2} \otimes |V\rangle_C) \\
 &= \frac{1}{2} (|H\rangle_A \otimes |H\rangle_{B_1} \otimes |H\rangle_{B_2} \otimes |H\rangle_C + |H\rangle_A \otimes |H\rangle_{B_1} \otimes |V\rangle_{B_2} \otimes |V\rangle_C + \\
 &\quad |V\rangle_A \otimes |V\rangle_{B_1} \otimes |H\rangle_{B_2} \otimes |H\rangle_C + |V\rangle_A \otimes |V\rangle_{B_1} \otimes |V\rangle_{B_2} \otimes |V\rangle_C) \\
 &= \frac{1}{2} (|\Phi^+\rangle_{B_1 B_2} \otimes |\Phi^+\rangle_{AC} + |\Phi^-\rangle_{B_1 B_2} \otimes |\Phi^-\rangle_{AC} + |\Psi^+\rangle_{B_1 B_2} \otimes |\Psi^+\rangle_{AC} + |\Psi^-\rangle_{B_1 B_2} \otimes |\Psi^-\rangle_{AC})
 \end{aligned}$$

Teleportation of the EPR state – Part 2

$$|\Psi\rangle = \frac{1}{2} \left(|\Phi^+\rangle_{B_1 B_2} \otimes |\Phi^+\rangle_{AC} + |\Phi^-\rangle_{B_1 B_2} \otimes |\Phi^-\rangle_{AC} + |\Psi^+\rangle_{B_1 B_2} \otimes |\Psi^+\rangle_{AC} + |\Psi^-\rangle_{B_1 B_2} \otimes |\Psi^-\rangle_{AC} \right)$$

- ▶ Bell measurements on B
 - ▶ 2-qubit measurements that detects Bell basis states
 - ▶ Examples: Detecting $|\Psi^\pm\rangle$ (creation-operator analysis)

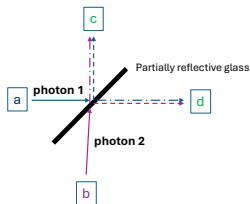


Figure: Beam splitter with polarizers at c and d

- ▶ $a^+ \rightarrow \frac{1}{\sqrt{2}}(c^+ + d^+)$; $b^+ \rightarrow \frac{1}{\sqrt{2}}(-c^+ + d^+)$ [parity phase]
- ▶ $|\Psi^+\rangle = \frac{1}{\sqrt{2}} (a_H^+ b_V^+ + a_V^+ b_H^+) |0\rangle \rightarrow \frac{1}{\sqrt{2}} (d_H^+ d_V^+ - c_H^+ c_V^+) |0\rangle$
- ▶ *Bunching*: Photons exit same port with opposite polarizations
- ▶ *Anti-bunching* $|\Psi^-\rangle \rightarrow \frac{1}{\sqrt{2}} (c_H^+ d_V^+ - d_H^+ c_V^+) |0\rangle$
- ▶ Harder to detect $|\Phi^\pm\rangle$. Requires non-linear optics.

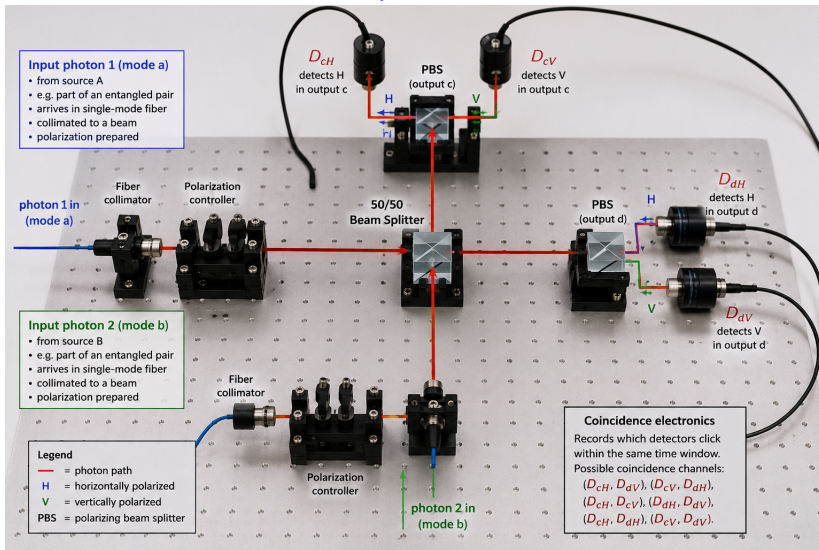


Figure: Beam splitter (chatgpt – with glitches)

Teleportation of the EPR state – Part 3

$$|\Psi\rangle = \frac{1}{2} \left(|\Phi^+\rangle_{B_1 B_2} \otimes |\Phi^+\rangle_{AC} + |\Phi^-\rangle_{B_1 B_2} \otimes |\Phi^-\rangle_{AC} + |\Psi^+\rangle_{B_1 B_2} \otimes |\Psi^+\rangle_{AC} + |\Psi^-\rangle_{B_1 B_2} \otimes |\Psi^-\rangle_{AC} \right)$$

- ▶ By measuring the Bell state on B, we entangle qubits on A and C
- ▶ E.g., if we measure $|\Psi^+\rangle_{B_1 B_2}$, then q_A and q_C are in state $|\Psi^+\rangle_{AC}$.
- ▶ However, we want to end up with the EPR state $|\Phi^+\rangle_{AC}$.
 - ▶ Send classical signal from B to C: “ $|\Psi^+\rangle_{B_1 B_2}$ was measured”
 - ▶ Based on information, pass C qubit through NOT gate

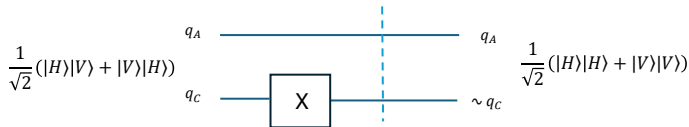


Figure: Converting $|\Psi^+\rangle_{AC}$ to EPR state

- ▶ Teleportation has less loss than direct transmission.
 - ▶ Assume A, B and C are equidistant
 - ▶ Direct AC transmission prob. is $\approx 1/4$ AB prob.
 - ▶ Direct AB transm. + entanglement-swapping has $\approx$ AB prob.

Teleportation of arbitrary states from A to D – Part 1

- ▶ First, establish and store an EPR pair between A and D
 - ▶ If A and D are very close, direct transmission may be OK
 - ▶ Otherwise, use entanglement swapping in a network
- ▶ Goal of teleportation:
 - ▶ Let $|\psi\rangle$ be the state of a photon on A.
 - ▶ $|\psi\rangle = \alpha|H\rangle + \beta|V\rangle$
 - ▶ We want to end up with a photon on B which is in state $|\psi\rangle$.
- ▶ Start with $|\psi\rangle$ -state photon and the EPR-state photon on A
 - ▶ $|\psi\rangle \rightarrow$ qubit-1, EPR $\rightarrow$ qubit-2 (node A), qubit-3 (node B)
 - ▶ Pass qubits-1,2 through a CNOT gate on node A
 - ▶ Then pass qubit-2 through a Hadamard gate on node A
 - ▶ Measure qubits-1,2 on node A
 - ▶ Finally send – classically – those measurements to B

Teleportation of arbitrary states from A to D – Part 2

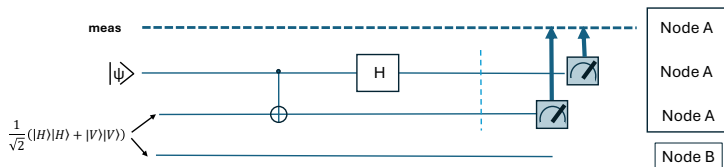


Figure: Entangle A's $|\psi\rangle$ photon with B's EPR photon

- ▶ Before the CNOT gate:

$$|\psi\rangle \otimes \left(\frac{|HH\rangle}{\sqrt{2}} + \frac{|VV\rangle}{\sqrt{2}} \right) = \frac{\alpha}{\sqrt{2}} |HHH\rangle + \frac{\alpha}{\sqrt{2}} |HVV\rangle + \frac{\beta}{\sqrt{2}} |VHH\rangle + \frac{\beta}{\sqrt{2}} |VVV\rangle$$

- ▶ After CNOT gate, before Hadamard gate:

$$\rightarrow \frac{\alpha}{\sqrt{2}} |HHH\rangle + \frac{\alpha}{\sqrt{2}} |HVV\rangle + \frac{\beta}{\sqrt{2}} |VVH\rangle + \frac{\beta}{\sqrt{2}} |VHV\rangle$$

- ▶ At the dotted vertical line:

$$\rightarrow \frac{\alpha}{2} |HHH\rangle + \frac{\beta}{2} |HHV\rangle + \frac{\alpha}{2} |HVV\rangle + \frac{\beta}{2} |HVH\rangle + \frac{\alpha}{2} |VHH\rangle - \frac{\beta}{2} |VHV\rangle + \frac{\alpha}{2} |VVV\rangle - \frac{\beta}{2} |VVH\rangle$$

- ▶ Then, on A, measure the first two qubits.
- ▶ Then classically communicate the result to D.

Teleportation of arbitrary states from A to D – Part 3

$$\rightarrow \frac{\alpha}{2}|HHH\rangle + \frac{\beta}{2}|HHV\rangle + \frac{\alpha}{2}|HVV\rangle + \frac{\beta}{2}|HVH\rangle + \frac{\alpha}{2}|VHH\rangle - \frac{\beta}{2}|VHV\rangle + \frac{\alpha}{2}|VVV\rangle - \frac{\beta}{2}|VVH\rangle$$

- ▶ Example: suppose the measurement on A results in state $|HH\rangle$.
 - ▶ Then the photon on D is in state $|D\rangle = \alpha|H\rangle + \beta|V\rangle$.
 - ▶ The overall normalization is set by $|\alpha|^2 + |\beta|^2 = 1$.
 - ▶ D knows the A-state was $|HH\rangle$
 - ▶ ...so does nothing to the D-photon.
 - ▶ $|D\rangle_{\text{final}} = |D\rangle = |\psi\rangle$.
- ▶ Example: suppose the measurement on A results in state $|HV\rangle$.
 - ▶ Then the photon on D is in state $|D\rangle = \alpha|V\rangle + \beta|H\rangle$.
 - ▶ D knows the A-state was $|HV\rangle$
 - ▶ ...so passes the D-photon through a NOT-gate.
 - ▶ $|D\rangle_{\text{final}} = U_{\text{NOT}}|D\rangle = \alpha|H\rangle + \beta|V\rangle = |\psi\rangle$
 - ▶ For A-measurements $|VH\rangle$ and $|VV\rangle$, D uses different gates
 - ▶ After passing through the gate, we have $|D\rangle_{\text{final}} = |\psi\rangle$.
- ▶ So we end up with a photon on D with state $|\psi\rangle$
 - ▶ Apart from transmission losses of the EPR pair, there are NO extra transmission losses for $|\psi\rangle$.

Concluding Thoughts

- ▶ Two complex numbers, α and β are “communicated” from A to D.
- ▶ But they are not transmitted
 - ▶ That information never appears anywhere between A and D
 - ▶ The only information transmitted is classical
 - ▶ One of 4 states measured at A (2 bits of information)
- ▶ So α and β can't be intercepted.
- ▶ REALLY WEIRD!
- ▶ However, it's not obvious how much information needs to be transmitted in order to communicate the amount of information encoded in α and β .
 - ▶ Remember that D measures $|\psi\rangle$ and obtains H or V .
 - ▶ Let D repeats the measurement N times
 - ▶ They see V about $|\beta|^2 N$ times.
 - ▶ But then A has to teleport $|\psi\rangle$ N times.
 - ▶ ..which requires $2N$ classical transmitted bits.
 - ▶ Only gives approximately $|\beta|$.
 - ▶ We need to compare bits of precision to probability curve