

Dark Matter

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The metric and energy-momentum (stress) tensor

- ▶ Observations and assumptions about our universe
 - ▶ Observe a universe which spatially homogeneous and isotropic.
 - ▶ We also observe that space is essentially flat.
 - ▶ We'll assume the universe evolves with time.
- ▶ Summarized by coordinates (t, r, θ, ϕ) and Robertson-Walker metric

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & a^2(t) & 0 & 0 \\ 0 & 0 & a^2(t)r^2 & 0 \\ 0 & 0 & 0 & a^2(t)r^2\sin^2\theta \end{pmatrix}$$

or more commonly shown as

$$ds^2 = -dt^2 + a^2(t)[dr^2 + r^2 d\Omega^2]$$

- ▶ At a large scale, model the universe as a perfect fluid.

$$T_{\mu\nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$

Equations of State

- ▶ Equation of state: $p = w\rho$. Three fluids of interest:
 - ▶ *Matter*: assume collisionless nonrelativistic particles (“dust”).

$$p_M = 0 \implies w = 0$$

- ▶ *Radiation*: Massless or almost massless particles (γ, ν).

$$\text{EM stress tensor gives } p_R = \frac{1}{3}\rho_R \implies w = \frac{1}{3}$$

- ▶ *Cosmological constant* Λ : A type of dark energy.

$$-\Lambda g_{\mu\nu} \implies p_\Lambda = -\rho_\Lambda \implies w = -1$$

- ▶ Stress tensor satisfies conservation of energy $0 = \nabla_\nu T_0^\mu$.

- ▶ Use the metric to obtain the covariant derivative.

$$\frac{\dot{\rho}}{\rho} = -3(1+w)\frac{\dot{a}}{a}$$

- ▶ Solve the differential equation for constant w

$$\rho \propto a^{-3(1+w)}$$

- ▶ The three fluids of interest:

- ▶ $\rho_M \propto a^{-3}$.

- ▶ $\rho_R \propto a^{-4}$.

- ▶ $\rho_\Lambda \propto a^0$. As the universe expands, this term wins out.

Friedmann Equations

- ▶ Einstein's equation (including cosmological constant) is

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi GT_{\mu\nu}.$$

- ▶ It's often convenient to rewrite this as

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi GT_{\mu\nu} - \Lambda g_{\mu\nu} \equiv 8\pi G\hat{T}_{\mu\nu},$$

where we've absorbed the cosmological term into the stress tensor.

- ▶ Note: even when a region is “empty”, $\hat{T}$ is non-zero
- ▶ Substitute the metric to obtain the Friedmann Equations
 - ▶ $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho.$
 - ▶ $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p).$

Solving the Friedmann Equations in our universe

Index each type of fluid by i (e.g. p_i)

- ▶ Plug in $p_i = w_i \rho_i$.
- ▶ Plug in $\rho_i \propto a_i^{-3(1+w_i)}$

Then use the redshift z instead of the scale factor a .

$$H \equiv \frac{\dot{a}}{a} = H_0 \sqrt{\Omega_M(1+z)^3 + \Omega_R(1+z)^4 + \Omega_\Lambda}$$

where measurements give $H_0 = 67.4 \pm 0.5 \text{ kms}^{-1} \text{ Mpc}^{-1}$, $\Omega_M \approx 0.31$, $\Omega_R \approx 0.00009$, and $\Omega_\Lambda \approx 0.685$.

Baryonic mass

- ▶ Estimates of mass density in these equations comes from
 - ▶ The Cosmic Microwave Background: Small amounts of anisotropy are shadows of pressure oscillations in the primordial universe. These oscillations are primarily from interactions of baryons with electrons and photons. Their frequency profiles predict baryon density in the early universe.
 - ▶ Big Bang Nucleosynthesis: Light elements including deuterium were formed and their abundance in the universe can be measured. Their relative abundance has to do with the speeds of nuclear reactions, which have to do with the baryon density in the early universe.
- ▶ The result is far less than ρ_M .
- ▶ We measure $\frac{\Omega_B}{\Omega_M} \approx 0.16$ where Ω_B is the baryon density.
- ▶ The non-baryonic mass is called **DARK MATTER**.

Katherine Freese “The Dark Side of the Universe”

At this point, turn to a presentation that covers other evidence of dark matter.

https:

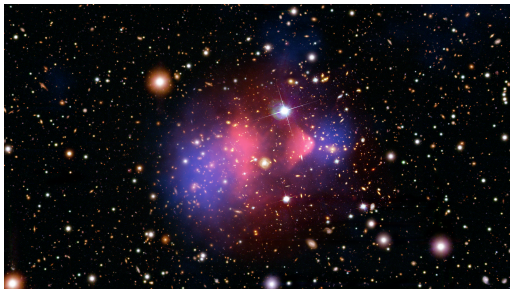
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The_Dark_Side_of_The_Universe_I.pdf?utm_source=chatgpt.com

Note – if you have printed out this presentation, it will be a lot harder to correctly enter this link than if you click on it from the original version before you downloaded it.

MOND

- ▶ Modified Newtonian Dynamics – Milgrom, 1983
- ▶ Explained rotation curves
 - ▶ When r is way larger than the radius of the solar system
 - ▶ $a \approx \sqrt{\frac{GM}{r^2}} a_0$ where a is acceleration
 - ▶ Then $\frac{v^2}{r} = \sqrt{\frac{GM}{r^2}} a_0 \implies v \approx \text{constant}$
 - ▶ But we wouldn't notice the difference in the solar system
- ▶ However, mis-predicts other phenomena, e.g. bullet cluster

Bullet cluster



- ▶ This observation is from the Chandra X-ray observatory
- ▶ Formed after the collision of 2 large clusters of galaxies
- ▶ Star-like objects are (baryonic) galaxies
- ▶ Pink blobs are hot gas originally in clusters
 - ▶ **Contains most baryonic matter**
 - ▶ On the right is bullet-shaped
- ▶ Blue blobs analyzed with lensing has much more matter
 - ▶ During collision, hot gas slows down through interactions